Inverse Reinforcement Learning

CS 285

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UC Berkeley



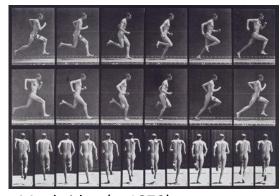
Today's Lecture

- 1. So far: manually design reward function to define a task
- 2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
- 3. Apply approximate optimality model from last time, but now learn the reward!

Goals:

- Understand the inverse reinforcement learning problem definition
- Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
- Understand a few practical inverse reinforcement learning algorithms we can use

Optimal Control as a Model of Human Behavior









Muybridge (c. 1870)

Mombaur et al. '09

Li & Todorov '06

Ziebart '08

$$\mathbf{a}_1, \dots, \mathbf{a}_T = \arg\max_{\mathbf{a}_1, \dots, \mathbf{a}_T} \sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

$$\pi = \arg\max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t)$$
optimize this to explain the data

Why should we worry about learning rewards?

The imitation learning perspective

Standard imitation learning:

- copy the actions performed by the expert
- no reasoning about outcomes of actions



Human imitation learning:

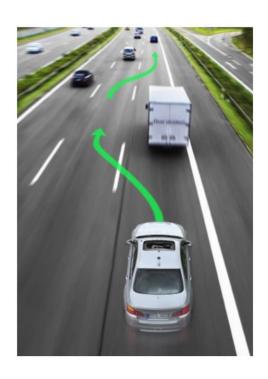
- copy the *intent* of the expert
- might take very different actions!



Why should we worry about learning rewards?

The reinforcement learning perspective

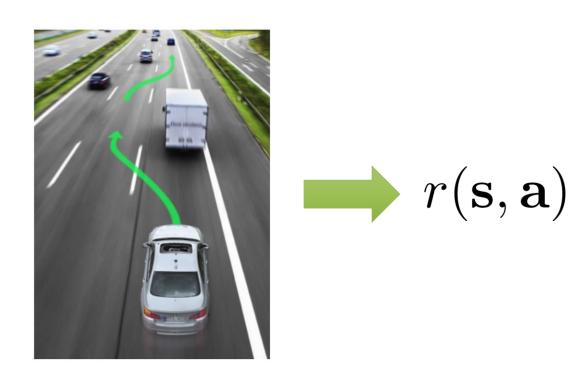




what is the reward?

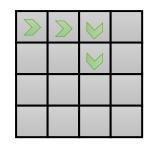
Inverse reinforcement learning

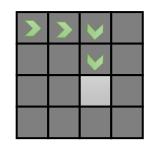
Infer reward functions from demonstrations

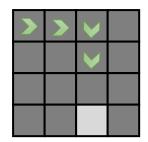


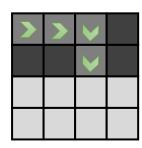
by itself, this is an **underspecified** problem

many reward functions can explain the **same** behavior









A bit more formally

"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ reward function $r(\mathbf{s}, \mathbf{a})$

learn $\pi^*(\mathbf{a}|\mathbf{s})$

inverse reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ samples $\{\tau_i\}$ sampled from $\pi^*(\tau)$

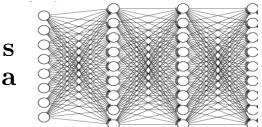
learn $r_{\psi}(\mathbf{s}, \mathbf{a})$ reward parameters

...and then use it to learn $\pi^*(\mathbf{a}|\mathbf{s})$

neural net reward function:

linear reward function:

$$r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$$



$$r_{\psi}(\mathbf{s}, \mathbf{a})$$
 parameters ψ

Feature matching IRL

linear reward function:

$$r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$$

if features **f** are important, what if we match their expectations?

let $\pi^{r_{\psi}}$ be the optimal policy for r_{ψ}

pick
$$\psi$$
 such that $E_{\pi^{r_{\psi}}}[\mathbf{f}(\mathbf{s}, \mathbf{a})] = E_{\pi^{\star}}[\mathbf{f}(\mathbf{s}, \mathbf{a})]$

state-action marginal under $\pi^{r_{\psi}}$

unknown optimal policy approximate using expert samples

maximum margin principle:

$$\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^*}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + m$$

need to somehow "weight" by similarity between π^* and π

still ambiguous!

Feature matching IRL & maximum margin

remember the "SVM trick":

$$\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^*}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + m$$

$$\min_{\psi} \frac{1}{2} \|\psi\|^2 \quad \text{ such that } \psi^T E_{\pi^*}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + D(\pi, \pi^*)$$

e.g., difference in feature expectations!

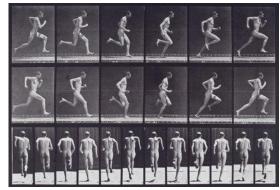
Issues:

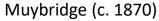
- Maximizing the margin is a bit arbitrary
- No clear model of expert suboptimality (can add slack variables...)
- Messy constrained optimization problem not great for deep learning!

Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

Optimal Control as a Model of Human Behavior







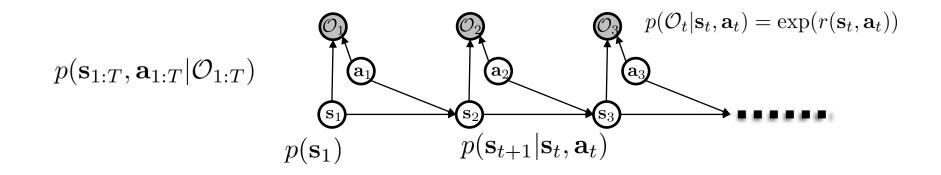
Mombaur et al. '09



Li & Todorov '06



Ziebart '08



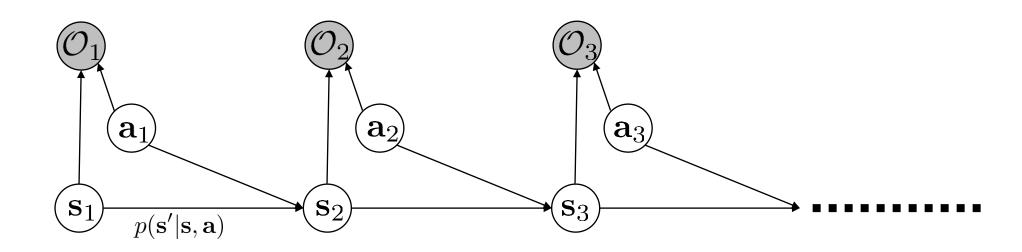
A probabilistic graphical model of decision making

$$p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T}) = ?? \quad \text{no assumption of optimal behavior!}$$

$$p(\tau | \mathcal{O}_{1:T}) \qquad p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

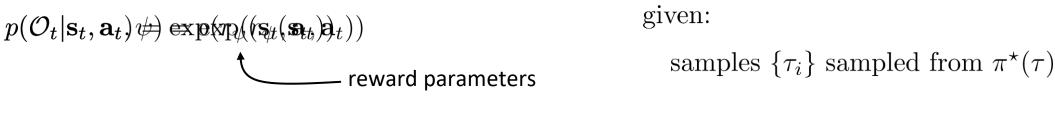
$$p(\tau | \mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$

$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$



Learning the Reward Function

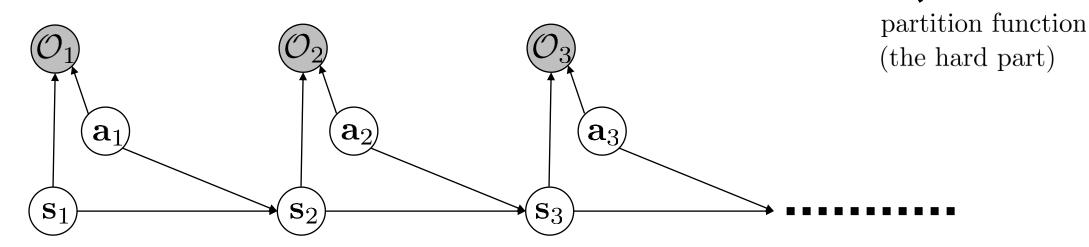
Learning the optimality variable



$$p(\tau|\mathcal{O}_{1:T}, \psi) \propto p(\tau) \exp\left(\sum_{t} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})\right)$$
 can ignore (independent of ψ)

maximum likelihood learning:

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} \log p(\tau_i | \mathcal{O}_{1:T}, \psi) = \max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z$$



The IRL partition function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z$$

$$Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

$$\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{Z} \int p(\tau) \exp(r_{\psi}(\tau)) \nabla_{\psi} r_{\psi}(\tau) d\tau$$
$$p(\tau | \mathcal{O}_{1:T}, \psi)$$

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

estimate with expert samples soft optimal policy under current reward

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)} \left[\nabla_{\psi} \sum_{t=1}^{T} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim p(\mathbf{s}_{t}, \mathbf{a}_{t}|\mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

$$p(\mathbf{a}_{t}|\mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t}|\mathcal{O}_{1:T}, \psi) \quad \text{where have we seen this before?}$$

$$= \frac{\beta(\mathbf{s}_{t}, \mathbf{a}_{t})}{\beta(\mathbf{s}_{t})} \qquad \propto \alpha(\mathbf{s}_{t})\beta(\mathbf{s}_{t})$$

$$p(\mathbf{a}_{t}|\mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t}|\mathcal{O}_{1:T}, \psi) \propto \beta(\mathbf{s}_{t}, \mathbf{a}_{t})\alpha(\mathbf{s}_{t})$$

backward message forward message

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)} [\nabla_{\psi} r_{\psi}(\tau)] \qquad \text{let } \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \propto \beta(\mathbf{s}_{t}, \mathbf{a}_{t}) \alpha(\mathbf{s}_{t})$$

$$\sum_{t=1}^{T} \int \int \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) d\mathbf{s}_{t} d\mathbf{a}_{t}$$

$$= \sum_{t=1}^{T} \vec{\mu}_{t}^{T} \nabla_{\psi} \vec{r}_{\psi}$$

state-action visitation probability for each $(\mathbf{s}_t, \mathbf{a}_t)$

The MaxEnt IRL algorithm

- 1. Given ψ , compute backward message $\beta(\mathbf{s}_t, \mathbf{a}_t)$ (see previous lecture)
- 2. Given ψ , compute forward message $\alpha(\mathbf{s}_t)$ (see previous lecture)
- 3. Compute $\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$
- 4. Evaluate $\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\psi} r_{\psi}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \sum_{t=1}^{T} \int \int \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) d\mathbf{s}_{t} d\mathbf{a}_{t}$
- 5. $\psi \leftarrow \psi + \eta \nabla_{\psi} \mathcal{L}$

Why MaxEnt?

in the case where $r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) = \psi^T \mathbf{f}(\mathbf{s}_t, \mathbf{a}_t)$, we can show that it optimizes

 $\max_{\psi} \mathcal{H}(\pi^{r_{\psi}}) \text{ such that } E_{\pi^{r_{\psi}}}[\mathbf{f}] = E_{\pi^{\star}}[\mathbf{f}]$ optimal max-ent policy under r^{ψ} unknown expert policy estimated with samples

as random as possible while matching features

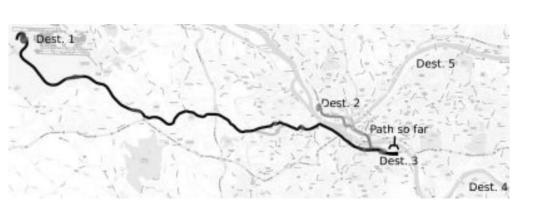
Ziebart et al. 2008: Maximum Entropy Inverse Reinforcement Learning

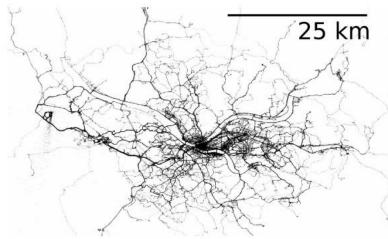
Maximum Entropy Inverse Reinforcement Learning

Brian D. Ziebart, Andrew Maas, J.Andrew Bagnell, and Anind K. Dey

School of Computer Science Carnegie Mellon University Pittsburgh, PA 15213

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Feature	Value
Highway	3.3 miles
Major Streets	2.0 miles
Local Streets	0.3 miles
Above 55mph	4.0 miles
35-54mph	1.1 miles
25-34 mph	0.5 miles
Below 24mph	0 miles
3+ Lanes	0.5 miles
2 Lanes	3.3 miles
1 Lane	1.8 miles

Feature	Value
Hard left turn	1
Soft left turn	3
Soft right turn	5
Hard right turn	0
No turn	25
U-turn	0

Approximations in High Dimensions

What's missing so far?

- MaxEnt IRL so far requires...
 - Solving for (soft) optimal policy in the inner loop
 - Enumerating all state-action tuples for visitation frequency and gradient
- To apply this in practical problem settings, we need to handle...
 - Large and continuous state and action spaces
 - States obtained via sampling only
 - Unknown dynamics

Unknown dynamics & large state/action spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

estimate with expert samples soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm then run this policy to sample $\{\tau_i\}$

$$J(\theta) = \sum_{t} E_{\pi(\mathbf{s}_{t}, \mathbf{a}_{t})} [r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})] + E_{\pi(\mathbf{s}_{t})} [\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_{t}))]$$

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{\psi} r_{\psi}(\tau_{j})$$

sum over expert samples sum over policy samples

More efficient sample-based updates

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{\psi} r_{\psi}(\tau_{j})$$

sum over expert samples sum over policy samples

improve learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm (a little) then run this policy to sample $\{\tau_i\}$

> looks expensive! what if we use "lazy" policy optimization? problem: estimator is now biased! wrong distribution! solution 1: use importance sampling

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{\sum_{j} w_j} \sum_{j=1}^{M} w_j \nabla_{\psi} r_{\psi}(\tau_j) \qquad w_j = \frac{p(\tau) \exp(r_{\psi}(\tau_j))}{\pi(\tau_j)}$$

Importance sampling

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{\sum_{j} w_{j}} \sum_{j=1}^{M} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})}$$

$$\frac{p(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \exp(r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{p(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \pi(\mathbf{a}_{t} | \mathbf{s}_{t})}$$

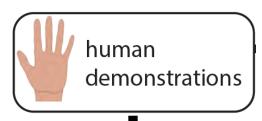
$$= \frac{\exp(\sum_{t} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\prod_{t} \pi(\mathbf{a}_{t} | \mathbf{s}_{t})}$$

each policy update w.r.t. r_{ψ} brings us closer to the target distribution!

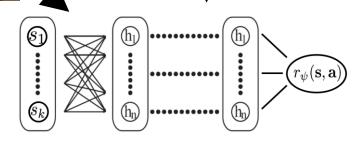
guided cost learning algorithm

(Finn et al. ICML '16)

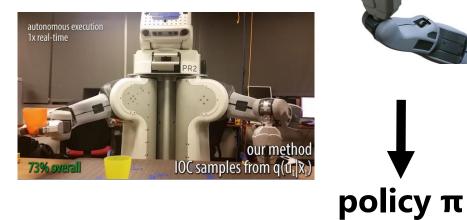




generate policy samples from π



Update reward using



samples & demos

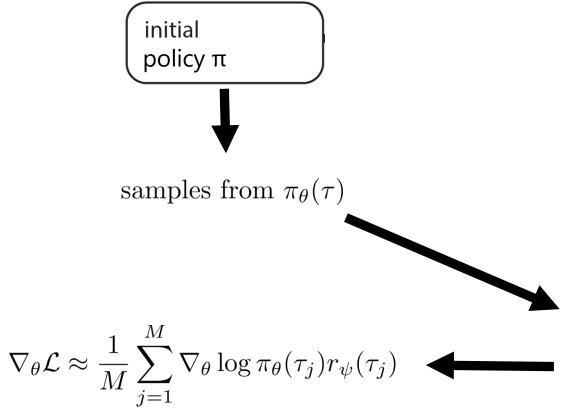
update π w.r.t. reward

reward r

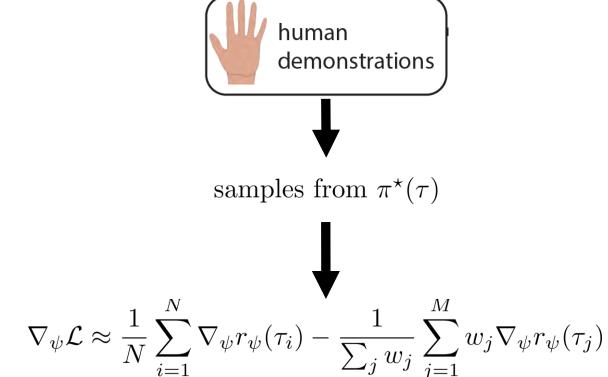
$$abla_{\psi} \mathcal{L} pprox rac{1}{N} \sum_{i=1}^{N}
abla_{\psi} r_{\psi}(au_i) - rac{1}{\sum_{j} w_j} \sum_{j=1}^{M} w_j
abla_{\psi} r_{\psi}(au_j)$$

IRL and GANs

It looks a bit like a game...



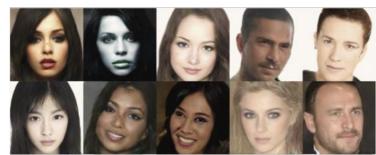
policy changed to make it harder to distinguish from demos

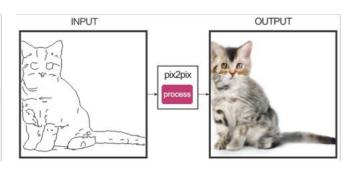


demos are made more likely, samples less likely

Generative Adversarial Networks



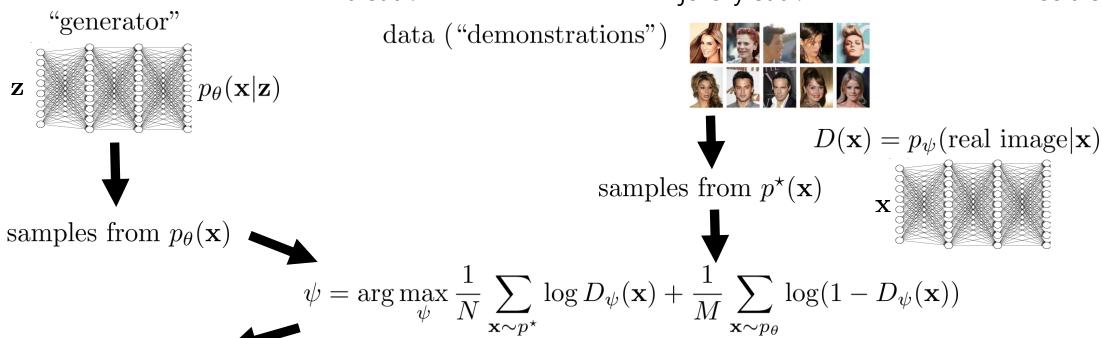




Zhu et al. '17

Arjovsky et al. '17

Isola et al. '17



 $\theta \leftarrow \arg\max_{\theta} E_{\mathbf{x} \sim p_{\theta}} \log D_{\psi}(\mathbf{x})$

Inverse RL as a GAN

$$D(\mathbf{x}) = p_{\psi}(\text{real image}|\mathbf{x})$$

which discriminator is best?

$$D^{\star}(\mathbf{x}) = \frac{p^{\star}(\mathbf{x})}{p_{\theta}(\mathbf{x}) + p^{\star}(\mathbf{x})}$$

for IRL, optimal policy approaches $\pi_{\theta}(\tau) \propto p(\tau) \exp(r_{\psi}(\tau))$

choose this parameterization for discriminator:

optimize Z w.r.t. same objective as $\psi!$

$$D_{\psi}(\tau) = \frac{p(\tau)\frac{1}{Z}\exp(r(\tau))}{p_{\theta}(\tau) + p(\tau)\frac{1}{Z}\exp(r(\tau))} = \frac{p(\tau)\frac{1}{Z}\exp(r(\tau))}{p(\tau)\prod_{t}\pi_{\theta}(\mathbf{a}_{t}|\mathbf{s}_{t}) + p(\tau)\frac{1}{Z}\exp(r(\tau))} = \frac{\frac{1}{Z}\exp(r(\tau))}{\prod_{t}\pi_{\theta}(\mathbf{a}_{t}|\mathbf{s}_{t}) + p(\tau)\frac{1}{Z}\exp(r(\tau))} = \frac{1}{Z}\exp(r(\tau))$$



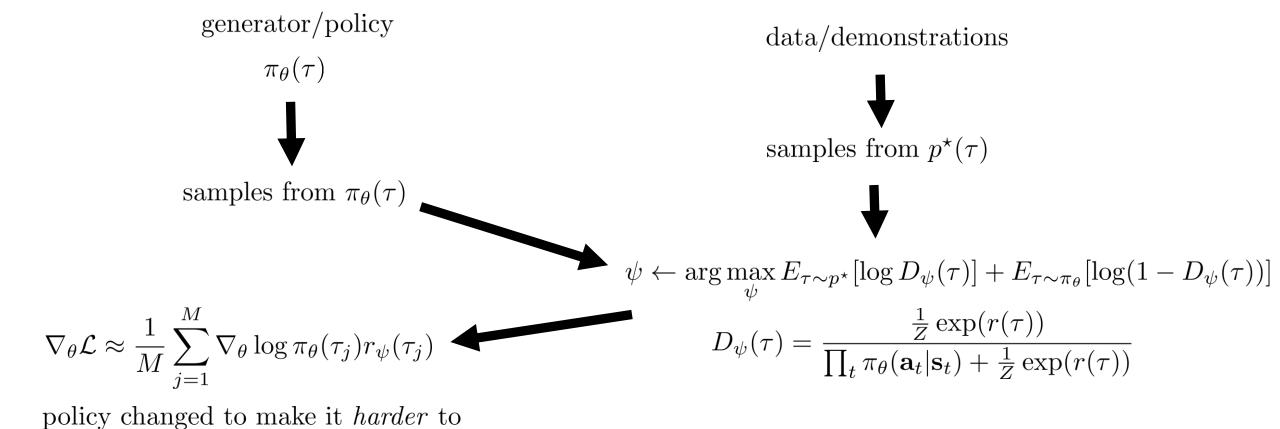
optimize this w.r.t. ψ

$$\psi \leftarrow \arg\max_{\psi} E_{\tau \sim p^*} [\log D_{\psi}(\tau)] + E_{\tau \sim \pi_{\theta}} [\log(1 - D_{\psi}(\tau))]$$

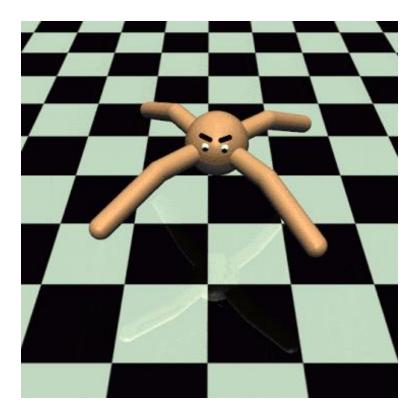
we don't need importance weights anymore – they are subsumed into Z

Inverse RL as a GAN

distinguish from demos



Generalization via inverse RL

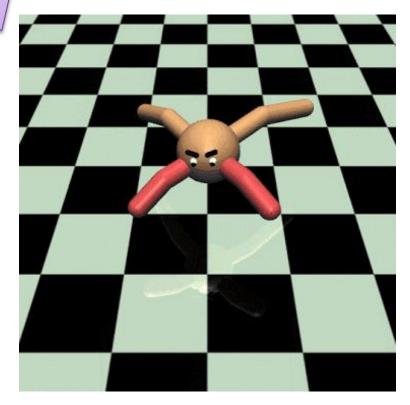


demonstration

what can we learn from the demonstration to enable better transfer?

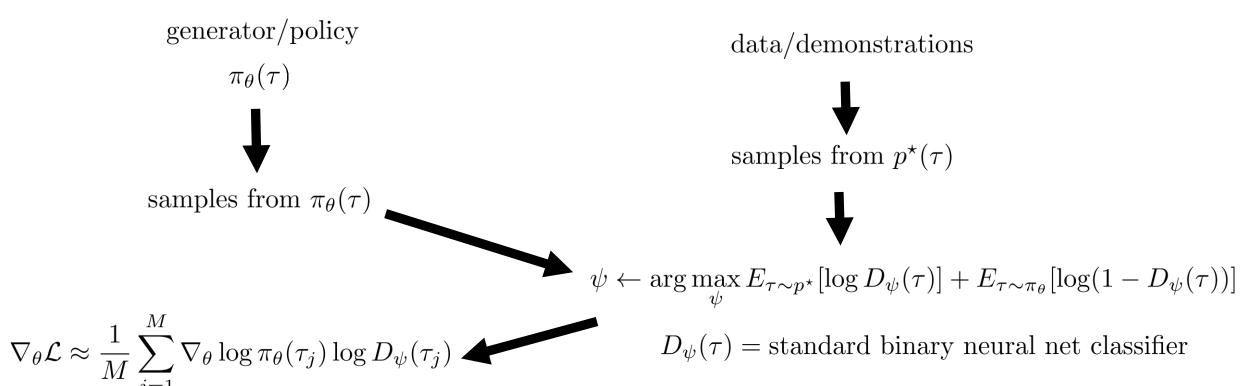
need to decouple the goal from the dynamics!

policy =
reward +
dynamics



reproduce behavior under different conditions

Can we just use a regular discriminator?



policy changed to make it harder to distinguish from demos

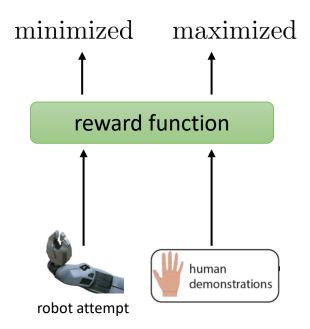
Pros & cons:

- + often simpler to set up optimization, fewer moving parts
- discriminator knows nothing at convergence
- generally cannot reoptimize the "reward"

Ho & Ermon. Generative adversarial imitation learning.

IRL as adversarial optimization

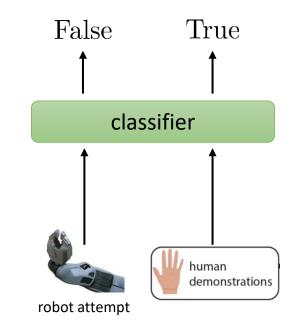
Guided Cost Learning Finn et al., ICML 2016



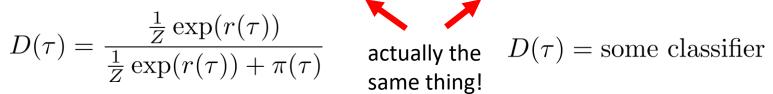
learns distribution $p(\tau)$ such that demos have max likelihood $p(\tau) \propto \exp(r(\tau))$ (MaxEnt model)

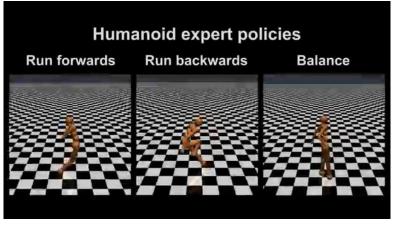
$$D(\tau) = \frac{\frac{1}{Z} \exp(r(\tau))}{\frac{1}{Z} \exp(r(\tau)) + \pi(\tau)}$$

Generative Adversarial Imitation Learning Ho & Ermon, NIPS 2016



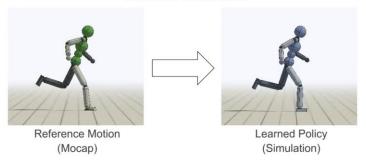
 $D(\tau) = \text{probability } \tau \text{ is a demo}$ use $\log D(\tau)$ as "reward"





Hausman, Chebotar, Schaal, Sukhatme, Lim

Motion Imitation



the goal is to train a simulated character to imitate the motion

Peng, Kanazawa, Toyer, Abbeel, Levine

Suggested Reading on Inverse RL

Classic Papers:

Abbeel & Ng ICML '04. Apprenticeship Learning via Inverse Reinforcement Learning. Good introduction to inverse reinforcement learning Ziebart et al. AAAI '08. Maximum Entropy Inverse Reinforcement Learning. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:

Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions

Wulfmeier et al. arXiv '16. Deep Maximum Entropy Inverse Reinforcement Learning.

MaxEnt inverse RL using deep reward functions

Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks

Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning