Inverse Reinforcement Learning

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Today's Lecture

- 1. So far: manually design reward function to define a task
- 2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
- 3. Apply approximate optimality model from last week, but now learn the reward!
- Goals:
 - Understand the inverse reinforcement learning problem definition
 - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
 - Understand a few practical inverse reinforcement learning algorithms we can use

Where does the reward function come from?



Mnih et al. '15

Real World Scenarios

robotics



autonomous driving



what is the reward? often use a proxy

frequently easier to provide expert data

Inverse reinforcement learning: infer reward function from roll-outs of expert policy

slides adapted from C. Finn

Why should we learn the reward?

Alternative: directly mimic the expert (behavior cloning)

- simply "ape" the expert's motions/actions
- doesn't necessarily capture the *salient* parts of the behavior
- what if the expert has different capabilities?

Can we reason about *what* the expert is trying to achieve instead?



slides adapted from C. Finn

Inverse Optimal Control / Inverse Reinforcement Learning:

infer reward function from demonstrations

(IOC/IRL) (Kalman '64, Ng & Russell '00)

given:

- state & action space
- samples from π^{\star}
- dynamics model (sometimes)

goal:

- recover reward function
- then use reward to get policy

Challenges

underdefined problem difficult to evaluate a learned reward demonstrations may not be precisely optimal



slides adapted from C. Finn

A bit more formally

"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ reward function $r(\mathbf{s}, \mathbf{a})$

learn $\pi^*(\mathbf{a}|\mathbf{s})$

inverse reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ samples $\{\tau_i\}$ sampled from $\pi^*(\tau)$

learn $r_{\psi}(\mathbf{s}, \mathbf{a})$ reward parameters

...and then use it to learn $\pi^{\star}(\mathbf{a}|\mathbf{s})$

neural net reward function:

linear reward function:

$$r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$$



Feature matching IRL

linear reward function: $r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$



still ambiguous!

if features \mathbf{f} are important, what if we match their expectations?

let $\pi^{r_{\psi}}$ be the optimal policy for r_{ψ}

pick ψ such that $E_{\pi^{r_{\psi}}}[\mathbf{f}(\mathbf{s}, \mathbf{a})] = E_{\pi^{\star}}[\mathbf{f}(\mathbf{s}, \mathbf{a})]$

state-action marginal under $\pi^{r_{\psi}}$

unknown optimal policy approximate using expert samples

maximum margin principle:

Feature matching IRL & maximum margin

remember the "SVM trick":

 $\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + m$ $\prod_{\psi} \frac{1}{2} \|\psi\|^2 \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + D(\pi,\pi^\star)$ e.g., difference in feature expectations!

Issues:

- Maximizing the margin is a bit arbitrary
- No clear model of expert suboptimality (can add slack variables...)
- Messy constrained optimization problem not great for deep learning!

Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

Optimal Control as a Model of Human Behavior



Muybridge (c. 1870)



Mombaur et al. '09





Li & Todorov '06

Ziebart '08





A probabilistic graphical model of decision making





Learning the optimality variable



The IRL partition function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z \qquad \qquad Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

$$\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_{\psi}(\tau)) \nabla_{\psi} r_{\psi}(\tau) d\tau$$

$$p(\tau | \mathcal{O}_{1:T}, \psi)$$

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

estimate with expert samples

soft optimal policy under current reward

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} \left[\nabla_{\psi} \sum_{t=1}^{T} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim p(\mathbf{s}_{t}, \mathbf{a}_{t} \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} \mid \mathcal{O}_{1:T}, \psi) \quad \text{where have we seen this before?}$$

$$= \frac{\beta(\mathbf{s}_{t}, \mathbf{a}_{t})}{\beta(\mathbf{s}_{t})} \qquad \propto \alpha(\mathbf{s}_{t})\beta(\mathbf{s}_{t})$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} \mid \mathcal{O}_{1:T}, \psi) \propto \beta(\mathbf{s}_{t}, \mathbf{a}_{t})\alpha(\mathbf{s}_{t})$$

$$backward message \qquad \text{forward message}$$

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$
$$\sum_{t=1}^{T} \int \int \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) d\mathbf{s}_{t} d\mathbf{a}_{t}$$
$$= \sum_{t=1}^{T} \vec{\mu}_{t}^{T} \nabla_{\psi} \vec{r}_{\psi}$$

let $\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$

state-action visitation probability for each $(\mathbf{s}_t, \mathbf{a}_t)$

The MaxEnt IRL algorithm

1. Given ψ , compute backward message $\beta(\mathbf{s}_t, \mathbf{a}_t)$ (see previous lecture)

2. Given ψ , compute forward message $\alpha(\mathbf{s}_t)$ (see previous lecture)

3. Compute
$$\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$$

4. Evaluate $\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\psi} r_{\psi}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - \sum_{t=1}^{T} \int \int \mu_t(\mathbf{s}_t, \mathbf{a}_t) \nabla_{\psi} r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_t d\mathbf{a}_t$ 5. $\psi \leftarrow \psi + \eta \nabla_{\psi} \mathcal{L}$

Why MaxEnt?

in the case where $r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) = \psi^T \mathbf{f}(\mathbf{s}_t, \mathbf{a}_t)$, we can show that it optimizes

 $\max_{\psi} \mathcal{H}(\pi^{r_{\psi}}) \text{ such that } E_{\pi^{r_{\psi}}}[\mathbf{f}] = E_{\pi^{\star}}[\mathbf{f}]$

optimal max-ent policy under r^{ψ}

unknown expert policy estimated with samples as random as possible while matching features

Ziebart et al. 2008: Maximum Entropy Inverse Reinforcement Learning

Case Study: MaxEnt IRL for road navigation MaxEnt IRL with hand-designed features for learning to navigate in urban environments based on taxi cab GPS data.

Maximum Entropy Inverse Reinforcement Learning

Brian D. Ziebart, Andrew Maas, J.Andrew Bagnell, and Anind K. Dey

School of Computer Science Carnegie Mellon University Pittsburgh, PA 15213

bziebart@cs.cmu.edu, amaas@andrew.cmu.edu, dbagnell@ri.cmu.edu, anind@cs.cmu.edu



Feature	Value	Γ	Feature	Value
Highway	3.3 miles		Hard left turn	1
Major Streets	2.0 miles		Soft left turn	3
Local Streets	0.3 miles		Soft right turn	5
Above 55mph	4.0 miles		Hard right turn	0
35-54mph	1.1 miles		No turn	25
25-34 mph	0.5 miles		U-turn	0
Below 24mph	0 miles	_		
3+ Lanes	0.5 miles			
2 Lanes	3.3 miles			
1 Lane	1.8 miles			

25 km

Break

What about larger RL problems?

- MaxEnt IRL: probabilistic framework for learning reward functions
- Computing gradient requires enumerating state-action visitations for all states and actions
 - Only really viable for small, discrete state and action spaces
 - Amounts to a dynamic programming algorithm (exact forwardbackward inference)
- For deep IRL, we want two things:
 - Large and continuous state and action spaces
 - Effective learning under unknown dynamics

Unknown dynamics & large state/action spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$\swarrow \qquad \qquad \checkmark$$
estimate with expert samples soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm then run this policy to sample $\{\tau_j\}$ $I(\theta) - \nabla$

$$J(\theta) = \sum_{t} E_{\pi(\mathbf{s}_{t},\mathbf{a}_{t})} [r_{\psi}(\mathbf{s}_{t},\mathbf{a}_{t})] + E_{\pi(\mathbf{s}_{t})} [\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_{t}))]$$

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{\psi} r_{\psi}(\tau_j)$$

sum over expert samples sum over policy samples

More efficient sample-based updates



sum over expert samples sum over policy samples

improve learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm (a little) then run this policy to sample $\{\tau_i\}$

looks expensive! what if we use "lazy" policy optimization?

problem: estimator is now biased! wrong distribution! solution: use importance sampling

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{\sum_j w_j} \sum_{j=1}^{M} w_j \nabla_{\psi} r_{\psi}(\tau_j) \qquad w_j = \frac{\exp(r_{\psi}(\tau_j))}{\pi(\tau_j)}$$

Importance sampling

$$\begin{split} \nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{\sum_{j} w_{j}} \sum_{j=1}^{M} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) & w_{j} = \frac{\exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})} \\ & \mathbf{v}_{j} = \frac{\exp(r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\pi(\tau_{j})} \\ & \mathbf{v}_{j} = \frac{\exp(r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\mathbf{v}_{j}(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \pi(\mathbf{a}_{t} | \mathbf{s}_{t})} \\ & \text{which sampling distribution } \pi(\tau) \text{ is best?} & = \frac{\exp(\sum_{t} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\prod_{t} \pi(\mathbf{a}_{t} | \mathbf{s}_{t})} \\ & \text{optimal IS distribution } q(x) \text{ for } E_{p(x)}[f(x)] \text{ is } q(x) \propto |f(x)| p(x) \\ & \text{ in our case, optimal } \pi \text{ is therefore } \pi(\tau) \propto \exp(r_{\psi}(\tau)) \end{split}$$

max-ent optimal policy for r_ψ

each policy update w.r.t. r_{ψ} brings us closer to the optimal distribution!



slides adapted from C. Finn

Example: learning pouring with a robot



Finn et al. Guided cost learning.

Example: learning pouring with a robot



Finn et al. Guided cost learning.

It looks a bit like a game...



distinguish from demos

Generative Adversarial Networks



Goodfellow et al. '14

Inverse RL as a GAN



which discriminator is best?

$$D^{\star}(\mathbf{x}) = \frac{p^{\star}(\mathbf{x})}{p_{\theta}(\mathbf{x}) + p^{\star}(\mathbf{x})}$$

for IRL, optimal policy approaches $\pi_{\theta}(\tau) \propto p(\tau) \exp(r_{\psi}(\tau))$



Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Inverse RL as a GAN



policy changed to make it *harder* to distinguish from demos

Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Generalization via inverse RL



demonstration

what can we learn from the demonstration to enable better transfer?

need to decouple the goal from the dynamics!

> policy = reward + dynamics



reproduce behavior under different conditions

Can we just use a regular discriminator?



policy changed to make it *harder* to distinguish from demos

- + often simpler to set up optimization, fewer moving parts
- discriminator knows *nothing* at convergence
- generally cannot reoptimize the "reward"

Ho & Ermon. Generative adversarial imitation learning.

IRL as adversarial optimization





Generative Adversarial Imitation Learning Ho & Ermon, NIPS 2016



 $D(\tau) =$ probability τ is a demo

use $\log D(\tau)$ as "reward"

same thing!



Hausman, Chebotar, Schaal, Sukhatme, Lim

Motion Imitation



the goal is to train a simulated character to imitate the motion.

Peng, Kanazawa, Toyer, Abbeel, Levine

Review

- IRL: infer unknown reward from expert demonstrations
- MaxEnt IRL: infer reward by learning under the control-as-inference framework
- MaxEnt IRL with dynamic programming: simple and efficient, but requires small state space and known dynamics
- Sampling-based MaxEnt IRL: generate samples to estimate the partition function
 - Guided cost learning algorithm
 - Connection to generative adversarial networks
 - Generative adversarial imitation learning (not IRL per se, but similar)

Suggested Reading on Inverse RL

Classic Papers:

Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:

Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning