Model-Based Reinforcement Learning

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

1. Project proposal due today!
2. Remember to start early on Homework 3!

Overview

1. Last lecture: choose good actions autonomously by backpropagating (or planning) through *known* system dynamics (e.g. known physics)
2. Today: what do we do if the dynamics are *unknown*?
   a. Fitting global dynamics models ("model-based RL")
   b. Fitting local dynamics models
3. Friday: learning dynamics for high-dimensional observations, such as images
4. Following Wednesday: combining optimal control and policy search to train neural network policies with the aid of optimal control

Today's Lecture

1. Overview of model-based RL
   - Learn only the model
   - Learn model & policy
2. What kind of models can we use?
3. Global models and local models
4. Learning with local models and trust regions

- Goals:
  - Understand the terminology and formalism of model-based RL
  - Understand the options for models we can use in model-based RL
  - Understand practical considerations of model learning
- Not much **deep** RL today, we'll see more advanced model-based RL later!

Why learn the model?

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

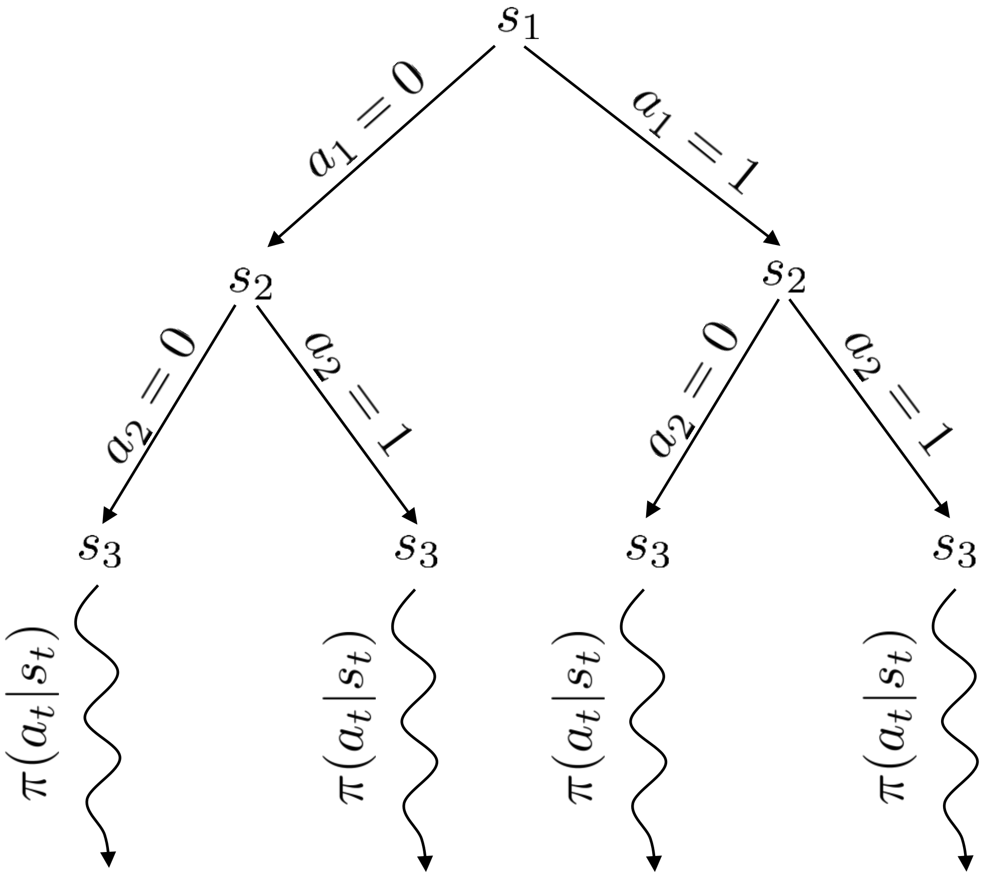
$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \dots + c(f(f(\dots)\dots), \mathbf{u}_T)$$

usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Why learn the model?

s_t a_t



Why learn the model?

If we knew $f(\mathbf{s}_t, \mathbf{a}_t) = \mathbf{s}_{t+1}$, we could use the tools from last week.

(or $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ in the stochastic case)

So let's learn $f(\mathbf{s}_t, \mathbf{a}_t)$ from data, and *then* plan through it!

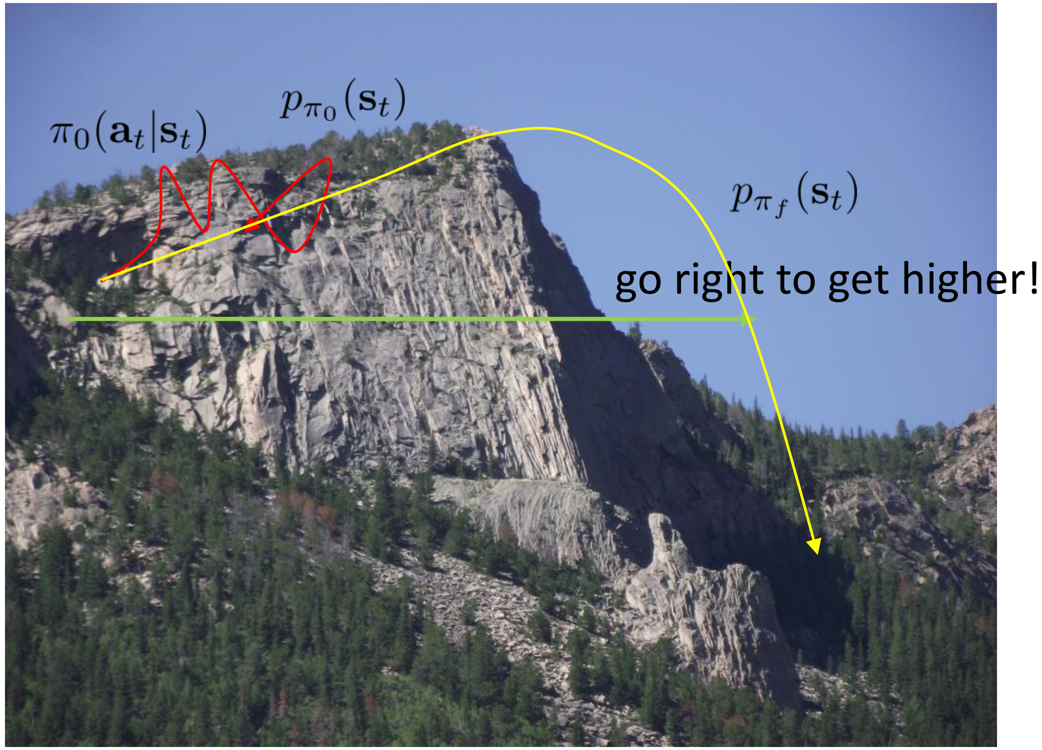
model-based reinforcement learning version 0.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

Does it work? Yes!

- Essentially how system identification works in classical robotics
- Some care should be taken to design a good base policy
- Particularly effective if we can hand-engineer a dynamics representation using our knowledge of physics, and fit just a few parameters

Does it work? No!



1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
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3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

- Distribution mismatch problem becomes exacerbated as we use more expressive model classes

Can we do better?

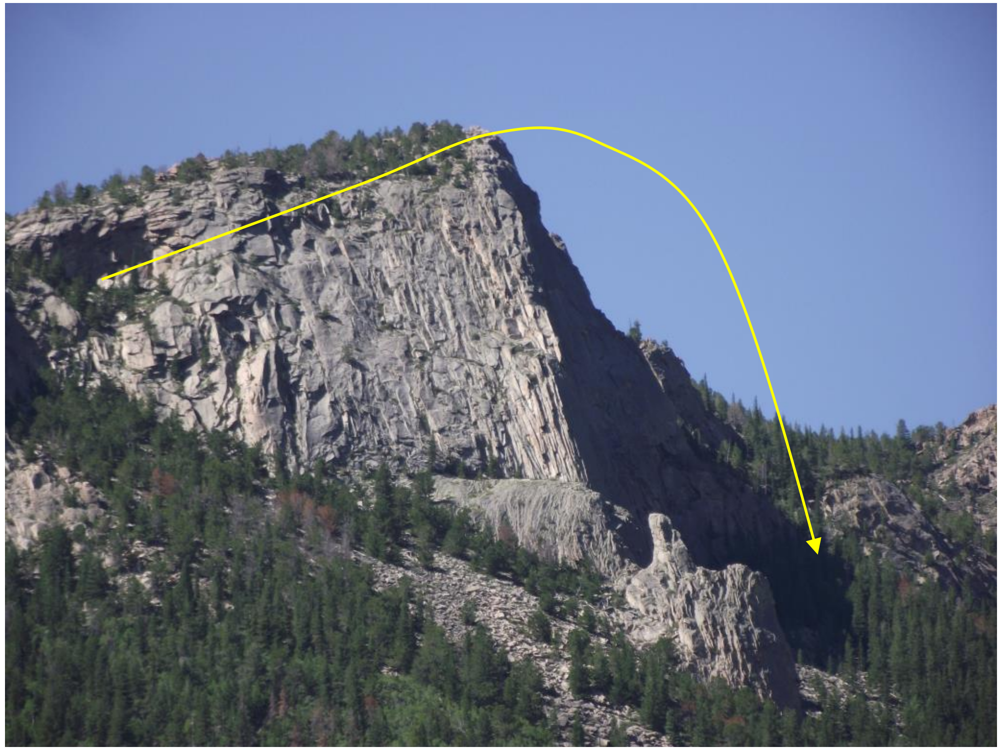
can we make $p_{\pi_0}(\mathbf{s}_t) = p_{\pi_f}(\mathbf{s}_t)$?

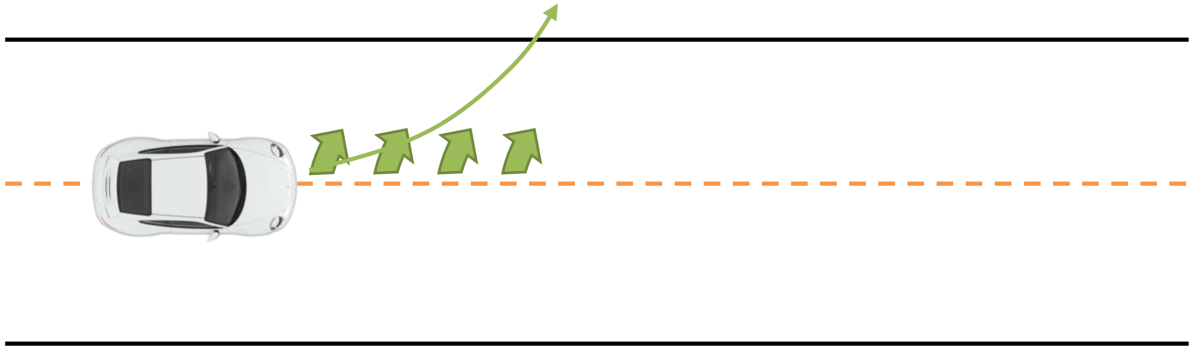
where have we seen that before? need to collect data from $p_{\pi_f}(\mathbf{s}_t)$

model-based reinforcement learning version 1.0:

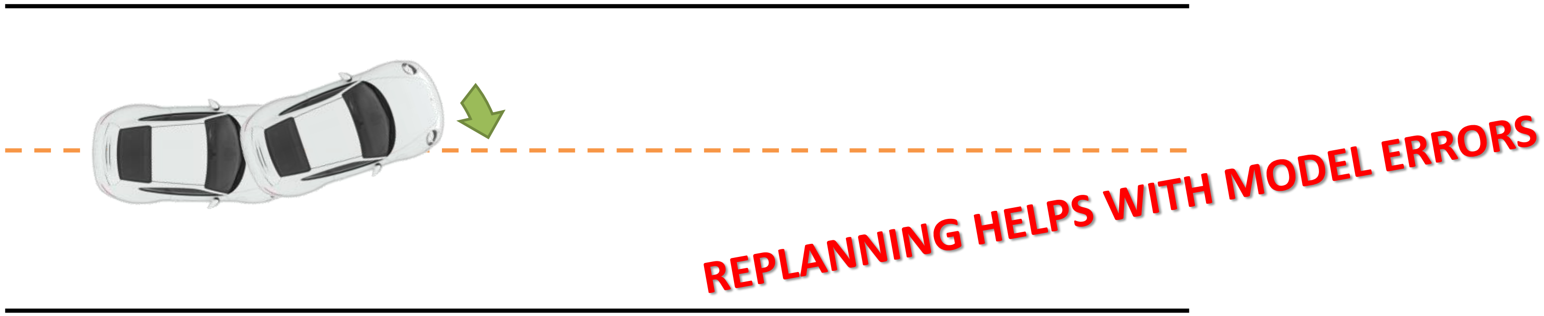
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to $\mathcal{D}$

What if we make a mistake?





Can we do better?



model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

This will be on HW4!

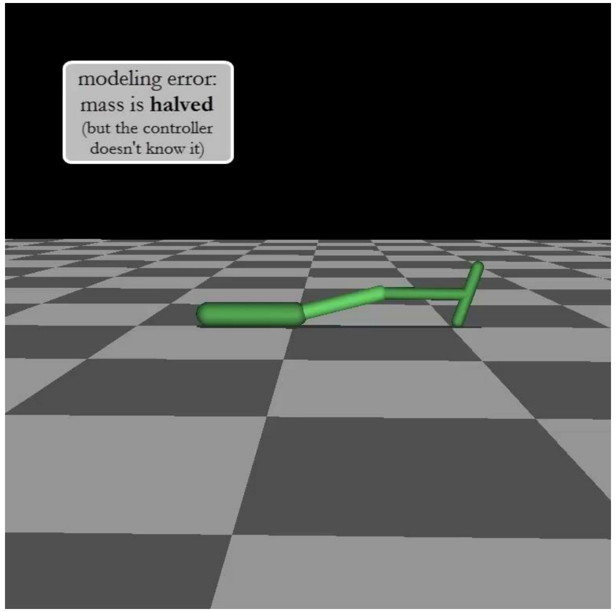
How to replan?

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

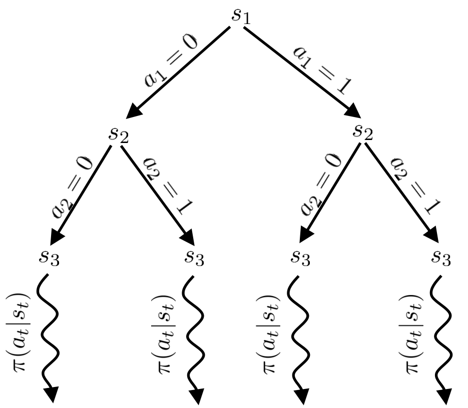
- The more you replan, the less perfect each individual plan needs to be
- Can use shorter horizons
- Even random sampling can often work well here!

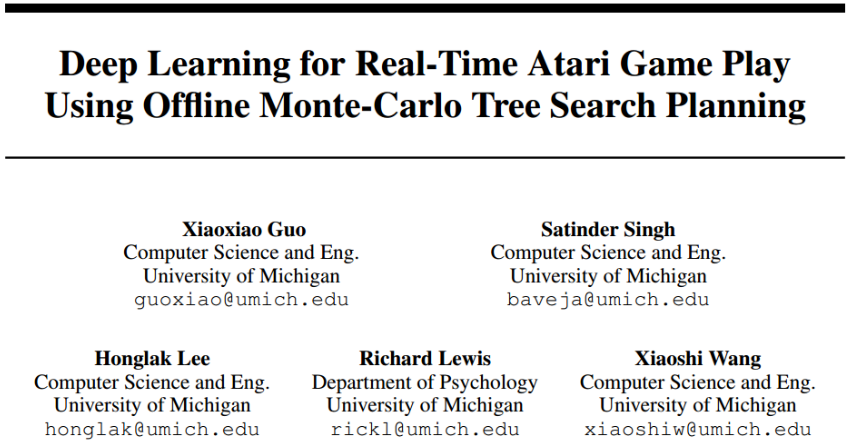


That seems like a lot of work...

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ to choose actions (e.g. using iLQR)
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$





Deep Learning for Real-Time Atari Game Play Using Offline Monte-Carlo Tree Search Planning

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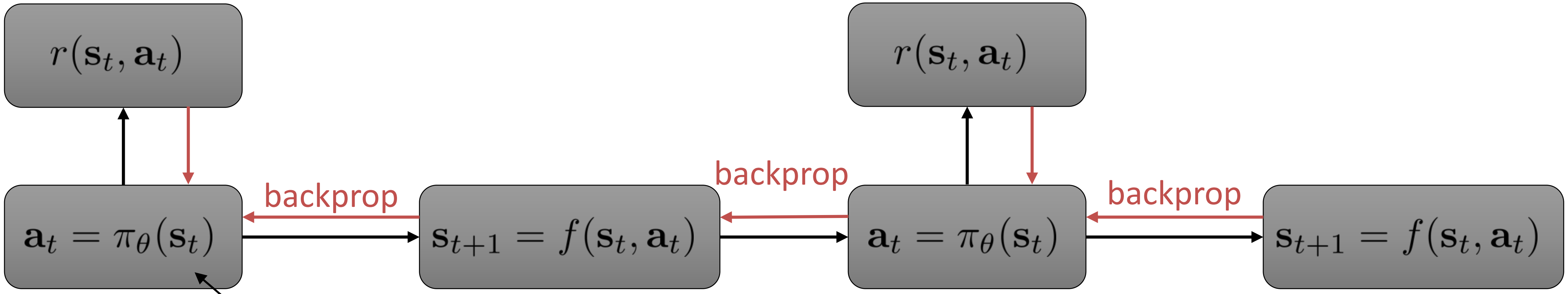
Backpropagate directly into the policy?



easy for deterministic policies, but also possible for stochastic policy (more on this later)

model-based reinforcement learning version 2.0:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

Summary

- Version 0.5: collect random samples, train dynamics, plan
  - Pro: simple, no iterative procedure
  - Con: distribution mismatch problem
- Version 1.0: iteratively collect data, replan, collect data
  - Pro: simple, solves distribution mismatch
  - Con: open loop plan might perform poorly, esp. in stochastic domains
- Version 1.5: iteratively collect data using MPC (replan at each step)
  - Pro: robust to small model errors
  - Con: computationally expensive, but have a planning algorithm available
- Version 2.0: backpropagate directly into policy
  - Pro: computationally cheap at runtime
  - Con: can be numerically unstable, especially in stochastic domains (more on this later)

Case study: model-based policy search with GPs

Learning to Control a Low-Cost Manipulator using Data-Efficient Reinforcement Learning

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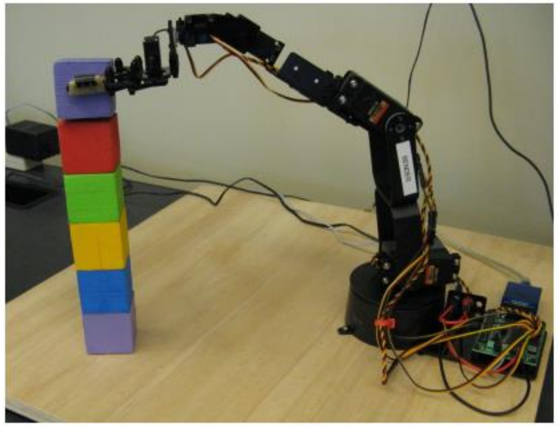

Case study: model-based policy search with GPs

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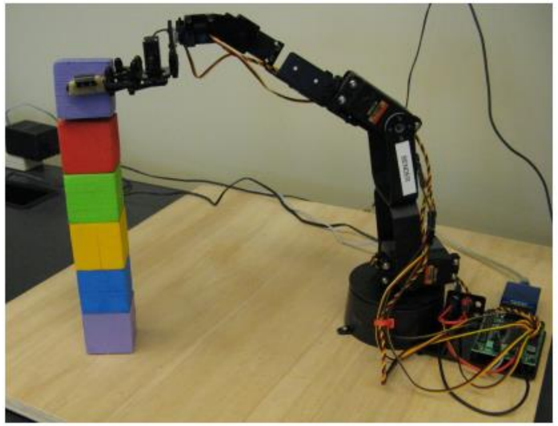


1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn GP dynamics model $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ to maximize $\sum_i \log p(\mathbf{s}'_i|\mathbf{s}_i, \mathbf{a}_i)$
3. backpropagate through $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

Case study: model-based policy search with GPs

3. backpropagate through $p(\mathbf{s}'|\mathbf{s},\mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

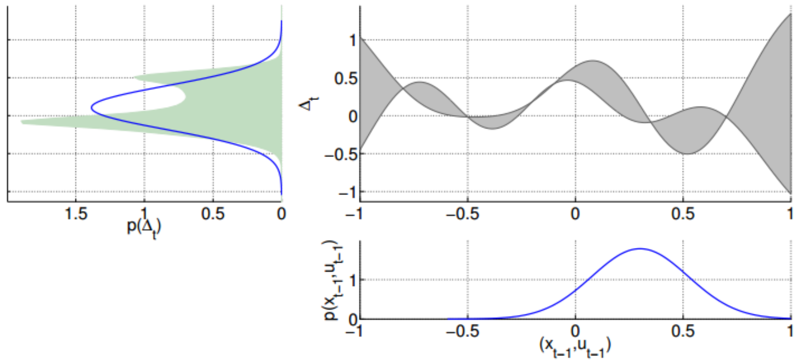
Given $p(\mathbf{s}_t)$, use $p(\mathbf{s}'|\mathbf{s},\mathbf{a})$ to compute $p(\mathbf{s}_{t+1})$

If $p(\mathbf{s}_t)$ is Gaussian, we can get a (non-Gaussian) $\bar{p}(\mathbf{s}_{t+1})$ in closed form

Project non-Gaussian $\bar{p}(\mathbf{s}_{t+1})$ to Gaussian $p(\mathbf{s}_{t+1})$ using moment matching

$E_{\mathbf{s}\sim p(\mathbf{s})}[c(\mathbf{s})]$ easy if c is nice and $p(\mathbf{s})$ Gaussian

Write $\sum_t E_{\mathbf{s}\sim p(\mathbf{s}_t)}[r(\mathbf{s}_t)]$ and differentiate

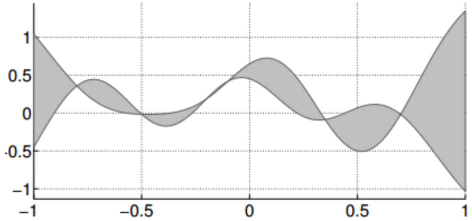


Marc Peter Deisenroth, Carl Edward Rasmussen, Dieter Fox

Learning to Control a Low-Cost Manipulator
using Data-efficient Reinforcement Learning

What kind of models can we use?

Gaussian process



GP with input $(\mathbf{s}, \mathbf{a})$ and output $\mathbf{s}'$

Pro: very data-efficient

Con: not great with non-smooth dynamics

Con: very slow when dataset is big

neural network

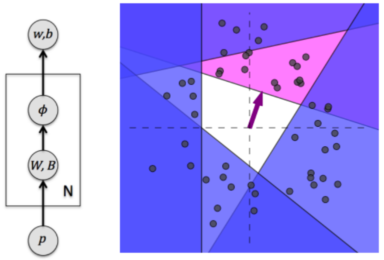


image: Punjani & Abbeel ‘14

Input is $(\mathbf{s}, \mathbf{a})$, output is $\mathbf{s}'$

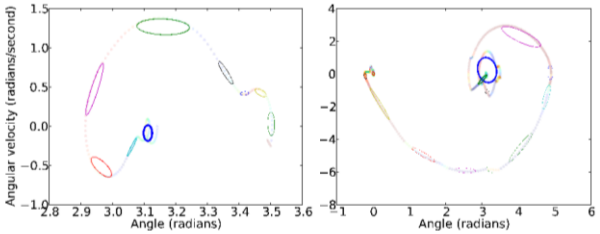
Euclidean training loss corresponds to Gaussian $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

More complex losses, e.g. output parameters of Gaussian mixture

Pro: very expressive, can use lots of data

Con: not so great in low data regimes

other



GMM over $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ tuples

Train on $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$, condition to get $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

For i^{th} mixture element, $p_i(\mathbf{s}, \mathbf{a})$ gives region where the mode $p_i(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ holds

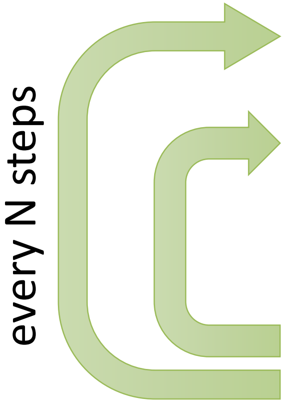
other classes: domain-specific models (e.g. physics parameters)

video prediction? more on this later in the course

Neural Network Dynamics for Model-Based Deep Reinforcement Learning with Model-Free Fine-Tuning

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$



2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions (random sampling)
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

These dynamics models are trained using trajectories that consist only of random steps.

At test time, we show that the models can be used to follow various desired trajectories.

Break

The trouble with global models

Global model: $f(\mathbf{s}_t, \mathbf{a}_t)$ represented by a big neural network

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to $\mathcal{D}$

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution

The trouble with global models

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution
- In some tasks, the model is much more complex than the policy

Local models

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\dots)\dots), \mathbf{u}_T)$$

usual story: differentiate via backpropagation and optimize!

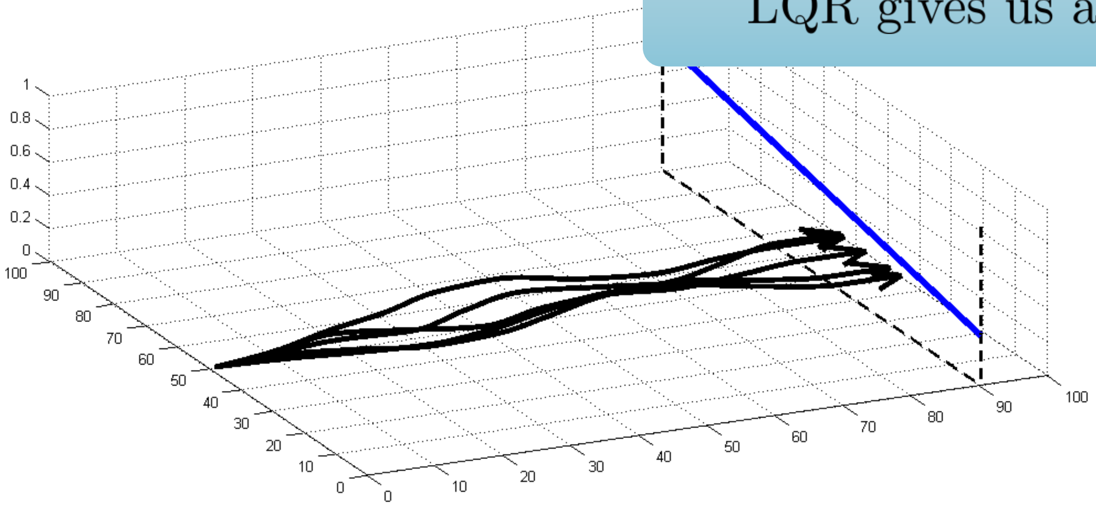
need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Local models

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

idea: just fit $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}$ around current trajectory or policy!

LQR gives us a linear feedback controller



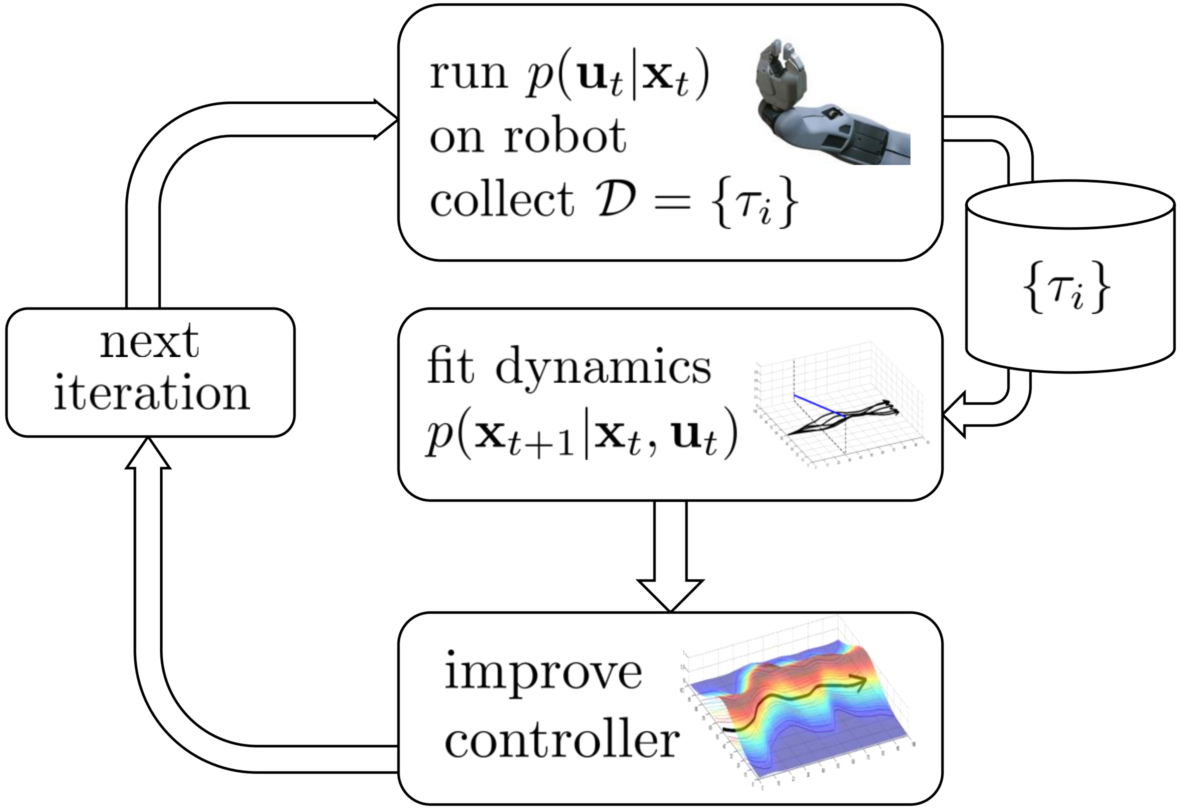
can **execute** in the real world!

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

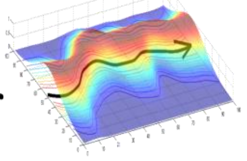
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



What controller to execute?

improve
controller



iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

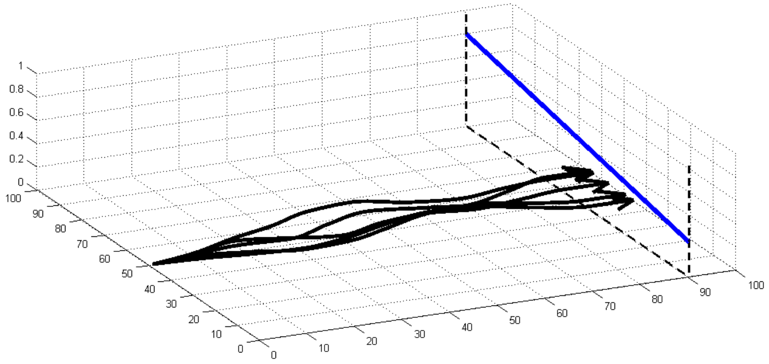
$$\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t$$

Version 0.5: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \hat{\mathbf{u}}_t)$

Doesn't correct deviations or drift

Version 1.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t)$

Better, but maybe a little too good?



Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Add noise so that all samples don't look the same!

What controller to execute?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Set $\Sigma_t = \mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}^{-1}$

$Q(\mathbf{x}_t, \mathbf{u}_t)$ is the cost to go: total cost we get after taking an action

$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix} + \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{q}_t$$

$\mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}$ is big if changing $\mathbf{u}_t$ changes the Q-value a lot!

If $\mathbf{u}_t$ changes Q-value a lot, don't vary $\mathbf{u}_t$ so much

Only act randomly when it minimally affects the cost to go

What controller to execute?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Set $\Sigma_t = \mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}^{-1}$

Standard LQR solves $\min \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t)$

Linear-Gaussian solution solves $\min \sum_{t=1}^{T} E_{(\mathbf{x}_t,\mathbf{u}_t)\sim p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

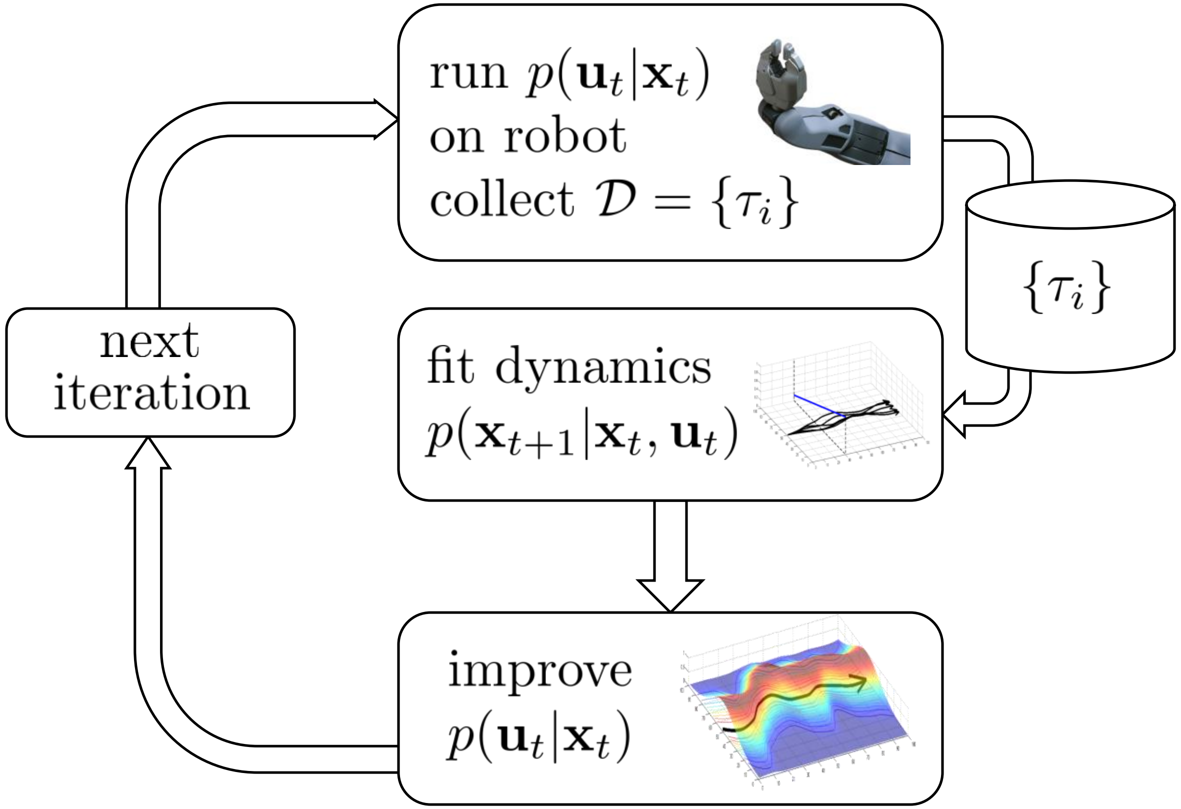
This is the *maximum entropy* solution: act as randomly as possible while minimizing cost

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

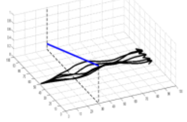
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



How to fit the dynamics?

fit dynamics
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$



$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

Version 1.0: fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

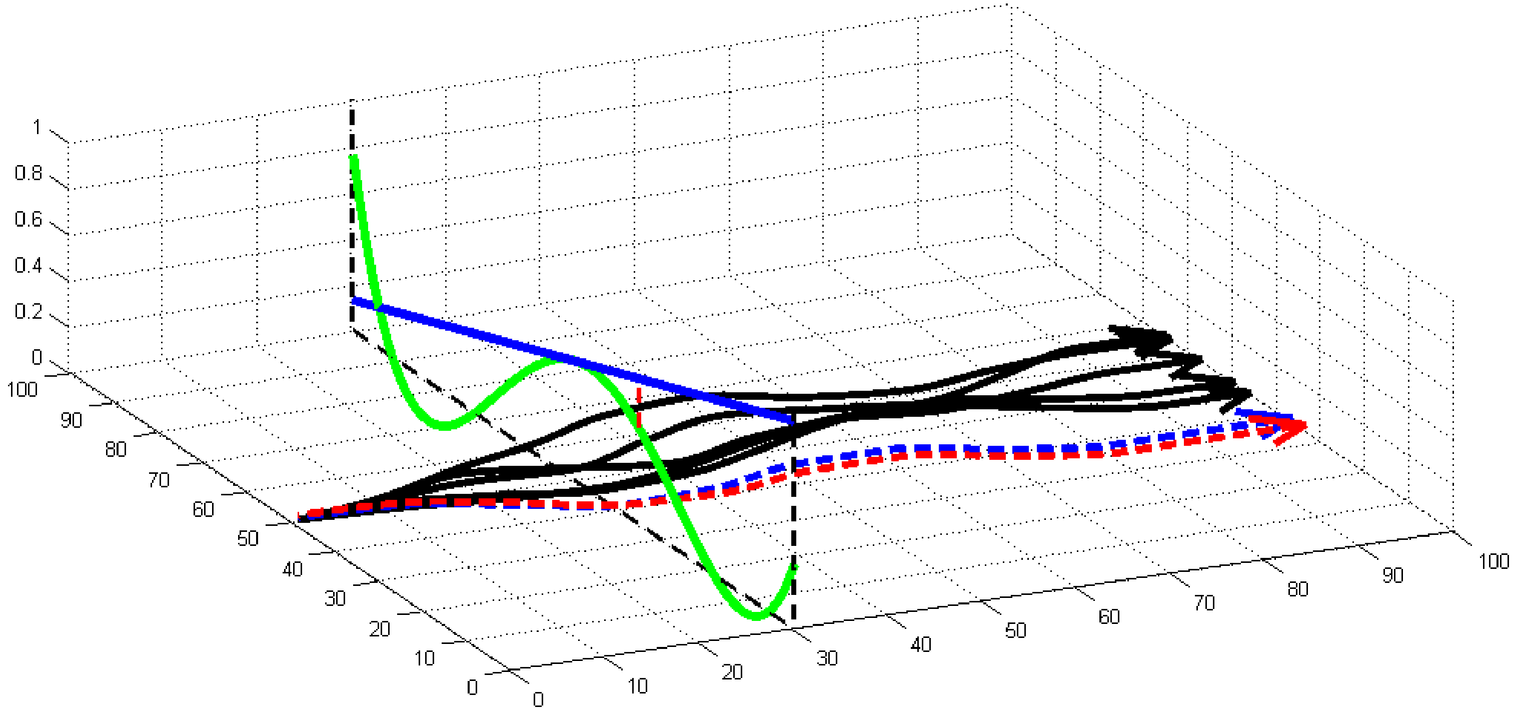
$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{df}{d\mathbf{x}_t} \quad \mathbf{B}_t \approx \frac{df}{d\mathbf{u}_t}$$

Can we do better?

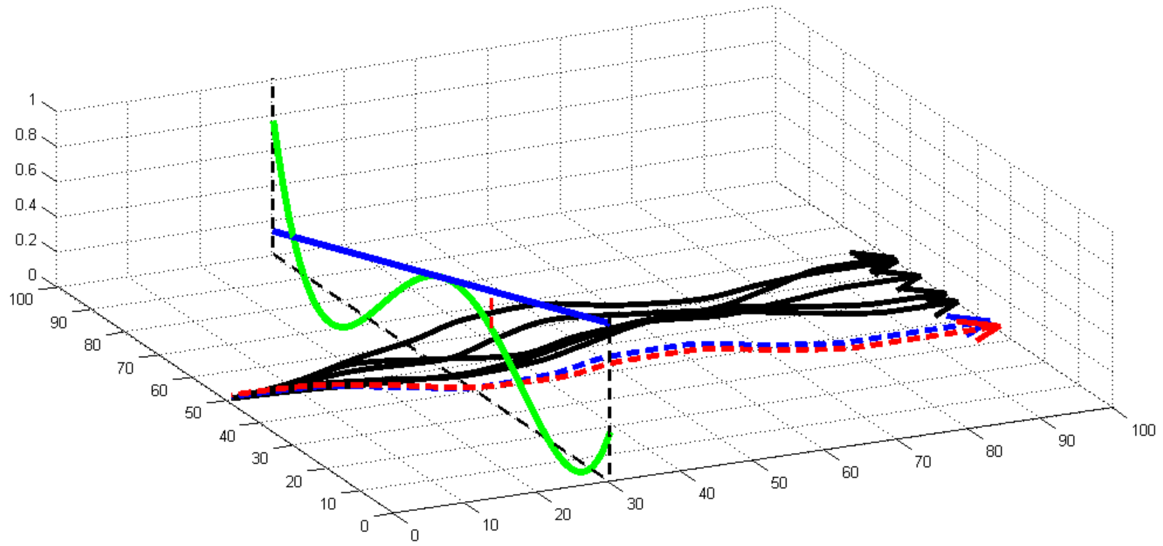
Version 2.0: fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ using *Bayesian* linear regression

Use your favorite *global* model as prior (GP, deep net, GMM)

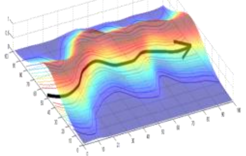
What if we go too far?



How to stay close to old controller?



improve $p(\mathbf{u}_t|\mathbf{x}_t)$



$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

$$p(\tau) = p(\mathbf{x}_1) \prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t) p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

What if the new $p(\tau)$ is "close" to the old one $\bar{p}(\tau)$?

If trajectory distribution is close, then dynamics will be close too!

What does "close" mean? $D_{\mathrm{KL}}(p(\tau) \| \bar{p}(\tau)) \leq \epsilon$

KL-divergences between trajectories

- Turns out to work very similarly to trust region for PG

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}[\log p(\tau) - \log \bar{p}(\tau)]$$

$$p(\tau) = p(\mathbf{x}_1) \prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t) p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) \qquad \bar{p}(\tau) = \underline{p(\mathbf{x}_1)} \prod_{t=1}^{T} \bar{p}(\mathbf{u}_t|\mathbf{x}_t) \underline{p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)}$$

dynamics & initial state are the same!

$$\log p(\tau) - \log \bar{p}(\tau) = \cancel{\log p(\mathbf{x}_1)} + \sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) + \cancel{\log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)}$$

$$\cancel{- \log p(\mathbf{x}_1)} + \sum_{t=1}^{T} - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) \cancel{- \log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)}$$

KL-divergences between trajectories

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}[\log p(\tau) - \log \bar{p}(\tau)]$$

$$\log p(\tau) - \log \bar{p}(\tau) = \log p(\mathbf{x}_1) + \sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) + \log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

$$- \log p(\mathbf{x}_1) + \sum_{t=1}^{T} - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}\left[\sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right]$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[\log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)]$$

KL-divergences between trajectories

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[\log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right]$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[- \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right] + E_{p(\mathbf{x}_t)}[\underbrace{E_{p(\mathbf{u}_t|\mathbf{x}_t)}[\log p(\mathbf{u}_t|\mathbf{x}_t)]}_{\text{negative entropy}}]$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[- \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))\right]$$

KL-divergences between trajectories

$$D_{\text{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$$

Reminder: Linear-Gaussian solves $\min \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

If we can get D_{KL} into the cost, we can just use iLQR!

But how?

We want a constraint: $D_{\text{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$

Digression: dual gradient descent

$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0$$

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \inf_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$\lambda \leftarrow \arg\max_{\lambda} g(\lambda)$$

how to maximize? Compute the gradient!

Digression: dual gradient descent

$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0 \qquad \mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \inf_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$g(\lambda) = \mathcal{L}(\mathbf{x}^\star(\lambda), \lambda)$$

$$\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\mathbf{x}^\star}\frac{d\mathbf{x}^\star}{d\lambda} + \frac{d\mathcal{L}}{d\lambda}$$

if $\mathbf{x}^\star = \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$, then $\frac{d\mathcal{L}}{d\mathbf{x}^\star} = 0$!

Digression: dual gradient descent

$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0$$

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \mathcal{L}(\mathbf{x}^\star(\lambda), \lambda)$$

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(\mathbf{x}^\star, \lambda)$$

1. Find $\mathbf{x}^\star \leftarrow \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$
2. Compute $\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(\mathbf{x}^\star, \lambda)$
3. $\lambda \leftarrow \lambda + \alpha \frac{dg}{d\lambda}$

DGD with iterative LQR

This is the constrained problem we want to solve:

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))\right]$$

$$\mathcal{L}(p,\lambda) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \lambda\mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))] - \lambda\epsilon$$

DGD with iterative LQR

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$$

$$\mathcal{L}(p,\lambda) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \lambda\mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))] - \lambda\epsilon$$

1. Find $p^\star \leftarrow \arg\min_p \mathcal{L}(p,\lambda)$
2. Compute $\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(p^\star,\lambda)$
3. $\lambda \leftarrow \lambda + \alpha\frac{dg}{d\lambda}$

this is the hard part, everything else is easy!

DGD with iterative LQR

1. Find $p^\star \leftarrow \arg\min_p \mathcal{L}(p, \lambda)$

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \lambda \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))] - \lambda\epsilon$$

Reminder: Linear-Gaussian solves $\min \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

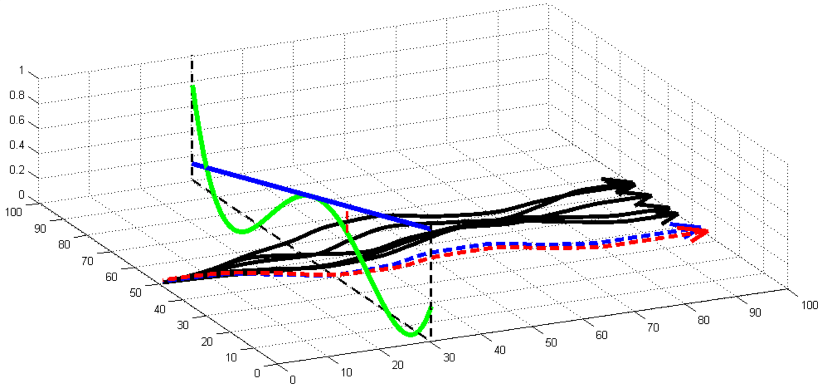
$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[\frac{1}{\lambda} c(\mathbf{x}_t, \mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))\right]$$

Just use LQR with cost $\tilde{c}(\mathbf{x}_t, \mathbf{u}_t) = \frac{1}{\lambda} c(\mathbf{x}_t, \mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)$

DGD with iterative LQR

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$$

1. Set $\tilde{c}(\mathbf{x}_t,\mathbf{u}_t) = \frac{1}{\lambda}c(\mathbf{x}_t,\mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)$
2. Use LQR to find $p^\star(\mathbf{u}_t|\mathbf{x}_t)$ using $\tilde{c}$
3. $\lambda \leftarrow \lambda + \alpha(D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) - \epsilon)$



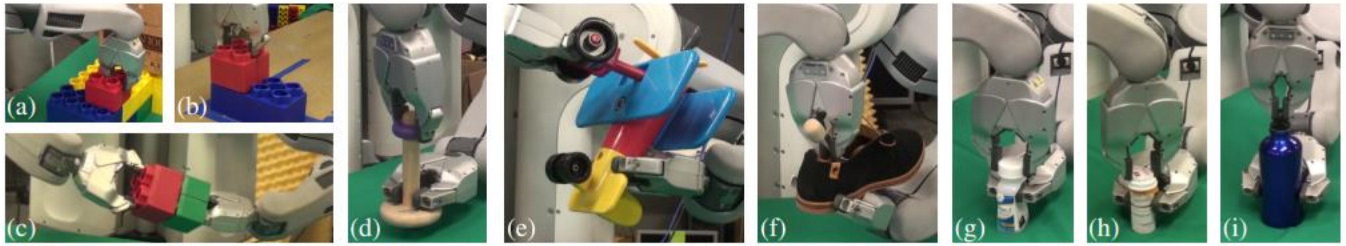
Trust regions & trajectory distributions

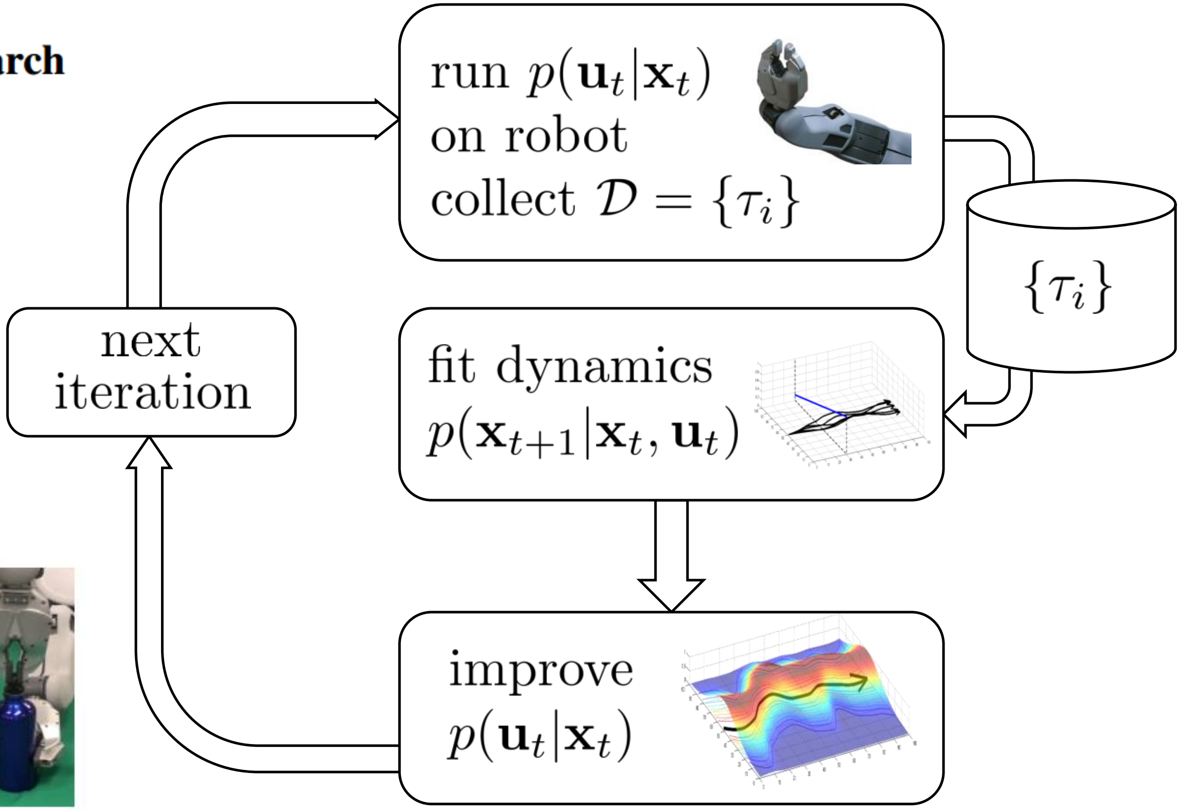
- Bounding KL-divergences between two policies or controllers, whether linear-Gaussian or more complex (e.g. neural networks) is really useful
- Bounding KL-divergence between policies is equivalent to bounding KL-divergences between trajectory distributions

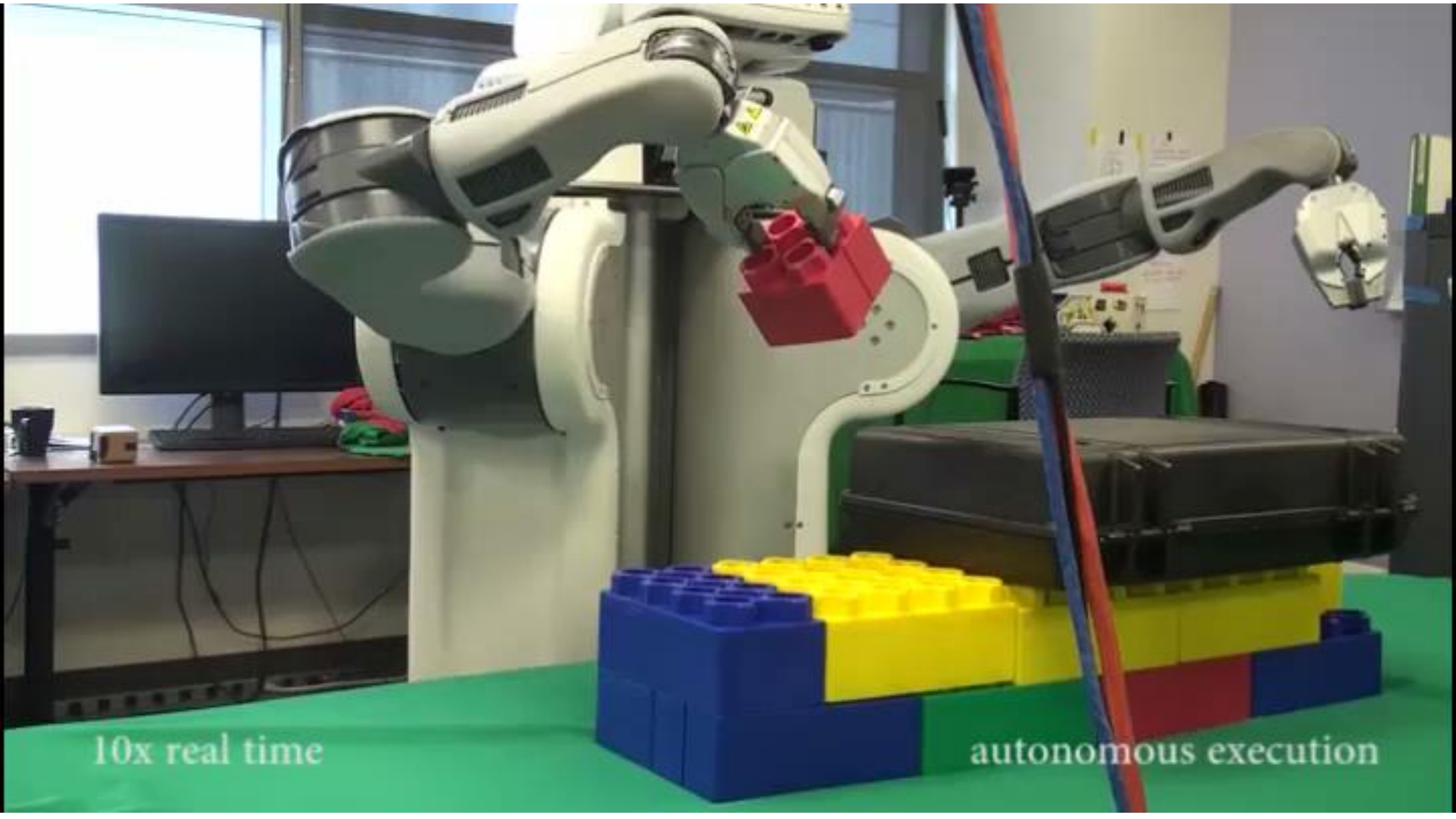
Example: local models & iterative LQR

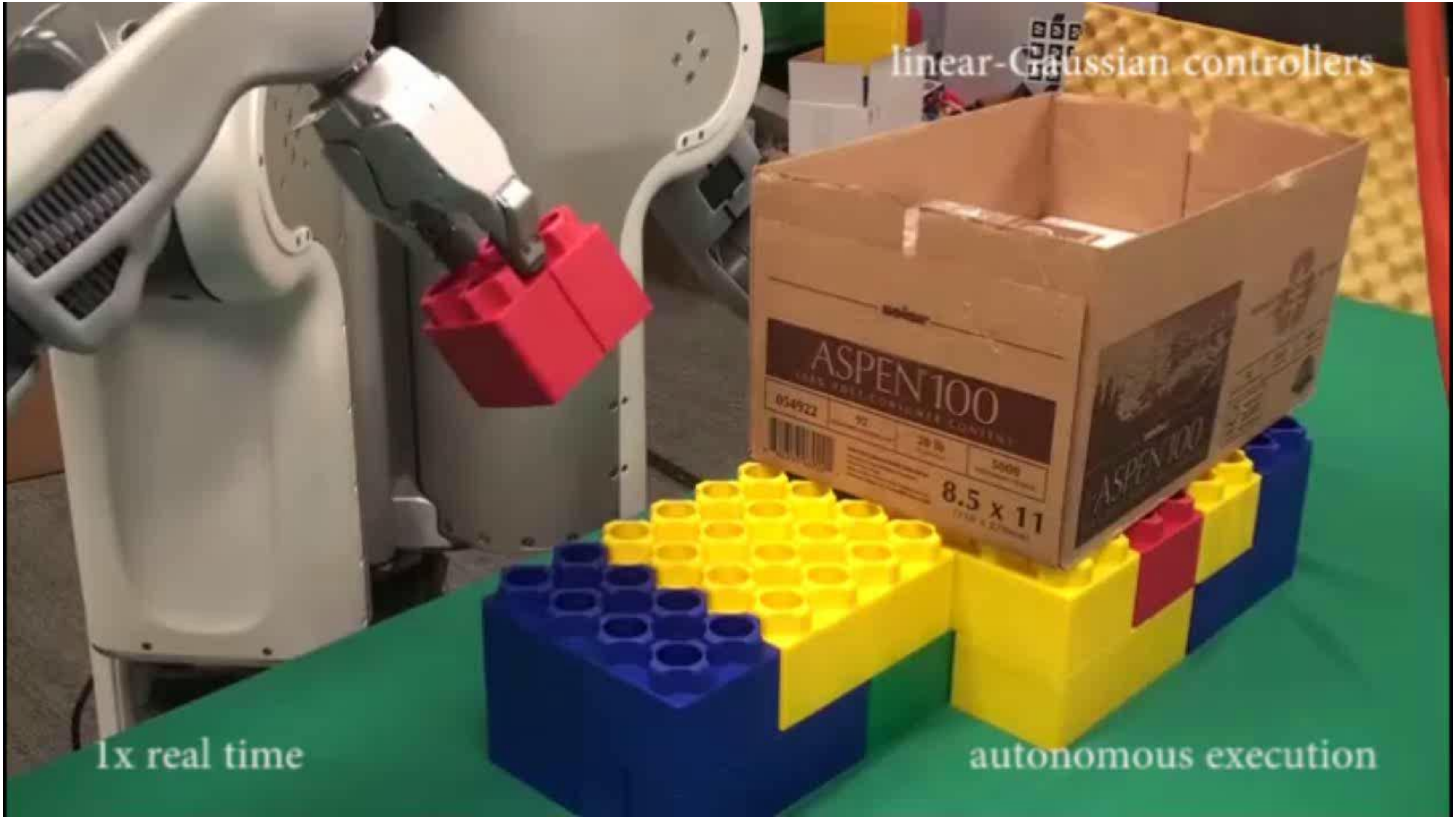
Learning Contact-Rich Manipulation Skills with Guided Policy Search

Sergey Levine, Nolan Wagener, Pieter Abbeel

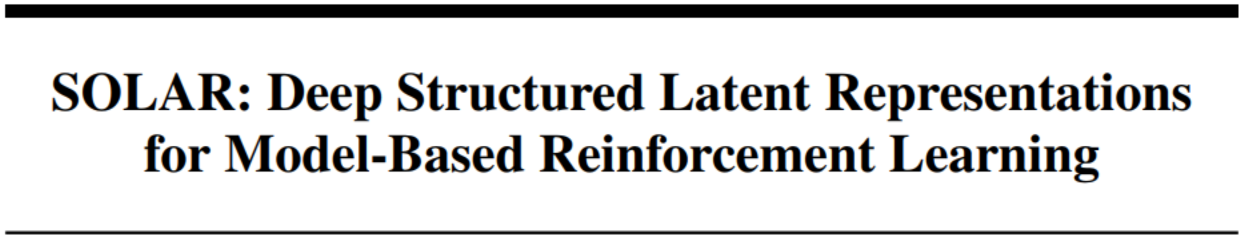





10x real time
autonomous execution


linear-Gaussian controllers
ASPEN 100
8.5 x 11
1x real time
autonomous execution

Example: local models with images


SOLAR: Deep Structured Latent Representations for Model-Based Reinforcement Learning