Inverse Reinforcement Learning

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

- 1. Project proposal due today at 11:59 pm!
- 2. Homework 4 due next week
- 3. After that, just the project left!

Today's Lecture

- 1. So far: manually design reward function to define a task
- 2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
- 3. Apply approximate optimality model from last week, but now learn the reward!
- Goals:
 - Understand the inverse reinforcement learning problem definition
 - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
 - Understand a few practical inverse reinforcement learning algorithms we can use

Where does the reward function come from?



Mnih et al. '15

Real World Scenarios

robotics



dialog autonomous driving

what is the reward? often use a proxy

frequently easier to provide expert data

Inverse reinforcement learning: infer reward function from roll-outs of expert policy

Why should we learn the reward?

Alternative: directly mimic the expert (behavior cloning)

- simply "ape" the expert's motions/actions
- doesn't necessarily capture the *salient* parts of the behavior
- what if the expert has different capabilities?

Can we reason about *what* the expert is trying to achieve instead?



Inverse Optimal Control / Inverse Reinforcement Learning:

infer reward function from demonstrations

(IOC/IRL) (Kalman '64, Ng & Russell '00)

given:

- state & action space
- samples from π^{\star}
- dynamics model (sometimes)

goal:

- recover reward function
- then use reward to get policy

Challenges

underdefined problem difficult to evaluate a learned reward demonstrations may not be precisely optimal



A bit more formally

"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ reward function $r(\mathbf{s}, \mathbf{a})$

learn $\pi^*(\mathbf{a}|\mathbf{s})$

inverse reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ samples $\{\tau_i\}$ sampled from $\pi^*(\tau)$

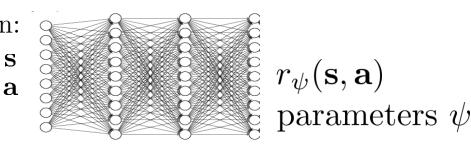
learn $r_{\psi}(\mathbf{s}, \mathbf{a})$ reward parameters

...and then use it to learn $\pi^*(\mathbf{a}|\mathbf{s})$

neural net reward function:

linear reward function:

$$r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$$



Feature matching IRL

linear reward function: $r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$



still ambiguous!

if features \mathbf{f} are important, what if we match their expectations?

let $\pi^{r_{\psi}}$ be the optimal policy for r_{ψ}

pick ψ such that $E_{\pi^{r_{\psi}}}[\mathbf{f}(\mathbf{s}, \mathbf{a})] = E_{\pi^{\star}}[\mathbf{f}(\mathbf{s}, \mathbf{a})]$

state-action marginal under $\pi^{r_{\psi}}$

unknown optimal policy approximate using expert samples

maximum margin principle:

Feature matching IRL & maximum margin

remember the "SVM trick":

 $\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + m$ $\prod_{\psi} \frac{1}{2} \|\psi\|^2 \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + D(\pi,\pi^\star)$ e.g., difference in feature expectations!

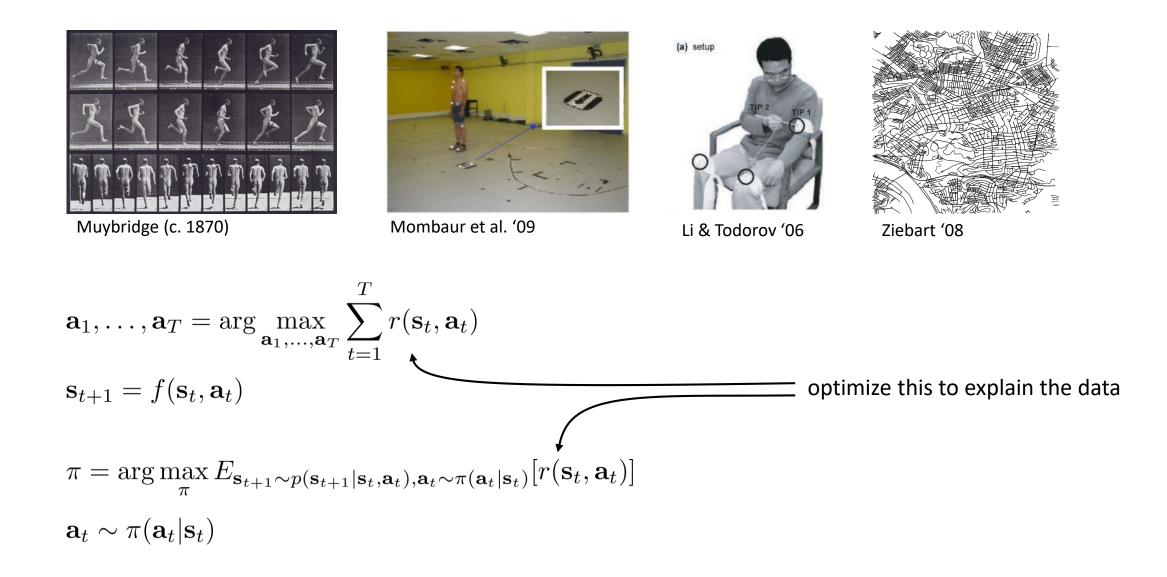
Issues:

- Maximizing the margin is a bit arbitrary
- No clear model of expert suboptimality (can add slack variables...)
- Messy constrained optimization problem not great for deep learning!

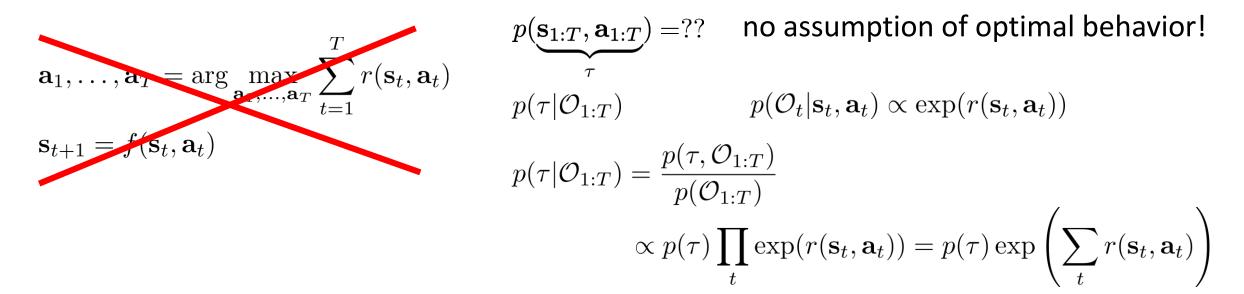
Further reading:

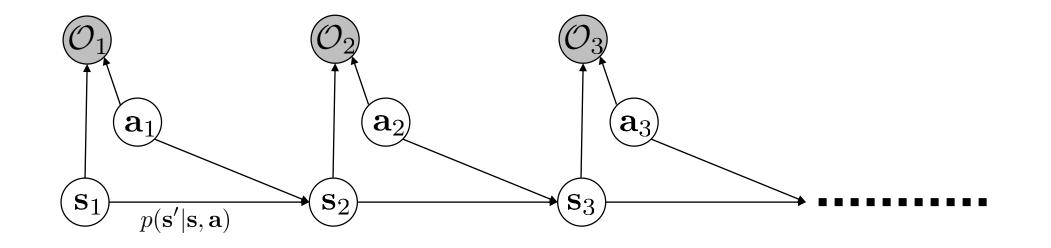
- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

Optimal Control as a Model of Human Behavior

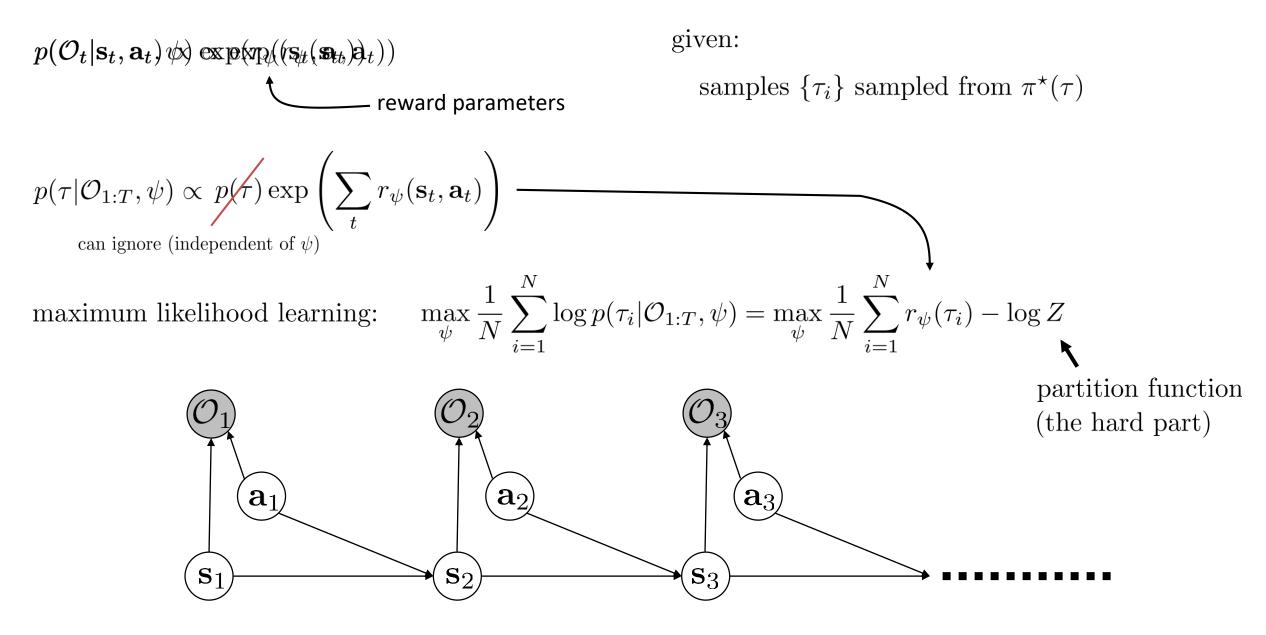


A probabilistic graphical model of decision making





Learning the optimality variable



The IRL partition function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z \qquad \qquad Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

$$\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_{\psi}(\tau)) \nabla_{\psi} r_{\psi}(\tau) d\tau$$

$$p(\tau | \mathcal{O}_{1:T}, \psi)$$

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

estimate with expert samples

soft optimal policy under current reward

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} \left[\nabla_{\psi} \sum_{t=1}^{T} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim p(\mathbf{s}_{t}, \mathbf{a}_{t} \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} \mid \mathcal{O}_{1:T}, \psi) \quad \text{where have we seen this before?}$$

$$= \frac{\beta(\mathbf{s}_{t}, \mathbf{a}_{t})}{\beta(\mathbf{s}_{t})} \qquad \propto \alpha(\mathbf{s}_{t})\beta(\mathbf{s}_{t})$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} \mid \mathcal{O}_{1:T}, \psi) \propto \beta(\mathbf{s}_{t}, \mathbf{a}_{t})\alpha(\mathbf{s}_{t})$$

$$backward message \qquad \text{forward message}$$

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$
$$\sum_{t=1}^{T} \int \int \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) d\mathbf{s}_{t} d\mathbf{a}_{t}$$
$$= \sum_{t=1}^{T} \vec{\mu}_{t}^{T} \nabla_{\psi} \vec{r}_{\psi}$$

let $\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$

state-action visitation probability for each $(\mathbf{s}_t, \mathbf{a}_t)$

The MaxEnt IRL algorithm

1. Given ψ , compute backward message $\beta(\mathbf{s}_t, \mathbf{a}_t)$ (see previous lecture)

2. Given ψ , compute forward message $\alpha(\mathbf{s}_t)$ (see previous lecture)

3. Compute
$$\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$$

4. Evaluate $\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\psi} r_{\psi}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - \sum_{t=1}^{T} \int \int \mu_t(\mathbf{s}_t, \mathbf{a}_t) \nabla_{\psi} r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_t d\mathbf{a}_t$ 5. $\psi \leftarrow \psi + \eta \nabla_{\psi} \mathcal{L}$

Why MaxEnt?

in the case where $r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) = \psi^T \mathbf{f}(\mathbf{s}_t, \mathbf{a}_t)$, we can show that it optimizes

 $\max_{\psi} \mathcal{H}(\pi^{r_{\psi}}) \text{ such that } E_{\pi^{r_{\psi}}}[\mathbf{f}] = E_{\pi^{\star}}[\mathbf{f}]$

optimal max-ent policy under r^{ψ}

unknown expert policy estimated with samples as random as possible while matching features

Ziebart et al. 2008: Maximum Entropy Inverse Reinforcement Learning

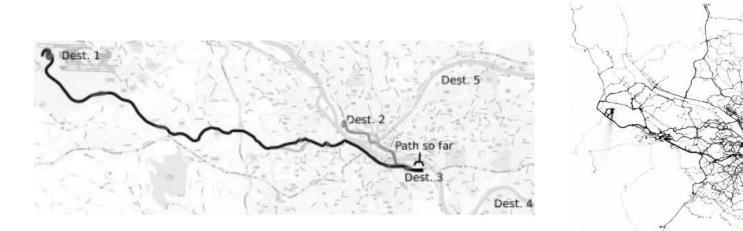
Case Study: MaxEnt IRL for road navigation MaxEnt IRL with hand-designed features for learning to navigate in urban environments based on taxi cab GPS data.

Maximum Entropy Inverse Reinforcement Learning

Brian D. Ziebart, Andrew Maas, J.Andrew Bagnell, and Anind K. Dey

School of Computer Science Carnegie Mellon University Pittsburgh, PA 15213

bziebart@cs.cmu.edu, amaas@andrew.cmu.edu, dbagnell@ri.cmu.edu, anind@cs.cmu.edu



Feature	Value	Feature	Value
Highway	3.3 miles	Hard left turn	1
Major Streets	2.0 miles	Soft left turn	3
Local Streets	0.3 miles	Soft right turn	5
Above 55mph	4.0 miles	Hard right turn	0
35-54mph	1.1 miles	No turn	25
25-34 mph	0.5 miles	U-turn	0
Below 24mph	0 miles		
3+ Lanes	0.5 miles		
2 Lanes	3.3 miles		
1 Lane	1.8 miles		

Case Study: MaxEnt Deep IRL

MaxEnt IRL with known dynamics (tabular setting), neural net cost

Maximum Entropy Deep Inverse Reinforcement Learning

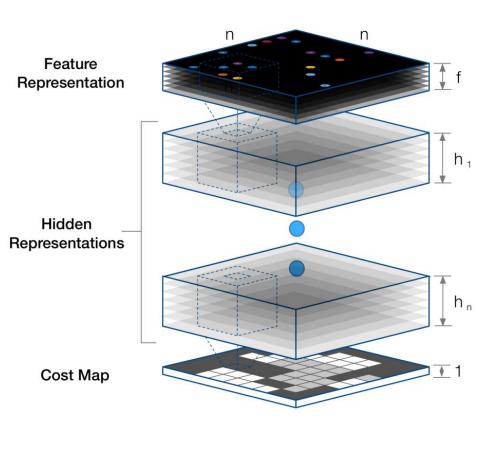
Markus WulfmeierMARKUS@ROBOTS.OX.AC.UKPeter OndrúškaONDRUSKA@ROBOTS.OX.AC.UKIngmar PosnerINGMAR@ROBOTS.OX.AC.UKMobile Robotics Group, Department of Engineering Science, University of OxfordIngmar Posner

NIPS Deep RL workshop 2015

Watch This: Scalable Cost-Function Learning for Path Planning in Urban Environments

Markus Wulfmeier¹, Dominic Zeng Wang¹ and Ingmar Posner¹ IROS 2016

Case Study: MaxEnt Deep IRL MaxEnt IRL with known dynamics (tabular setting), neural net cost



Algorithm 1 Maximum Entropy Deep IRL

Input: $\mu_D^a, f, S, A, T, \gamma$

Output: optimal weights θ^*

1: $\theta^1 = \text{initialise_weights}()$

Iterative model refinement

- 2: for n = 1 : N do
- 3: $r^n = \operatorname{nn}_{forward}(f, \theta^n)$

Solution of MDP with current reward

- 4: $\pi^n = \operatorname{approx_value_iteration}(r^n, S, A, T, \gamma)$
- 5: $\mathbb{E}[\mu^n] = \text{propagate_policy}(\pi^n, S, A, T)$

Determine Maximum Entropy loss and gradients

- 6: $\mathcal{L}_D^n = \log(\pi^n) \times \mu_D^a$
- 7: $\frac{\partial \mathcal{L}_D^n}{\partial r^n} = \mu_D \mathbb{E}[\mu^n]$

$\begin{array}{l} \textbf{Compute network gradients}\\ \frac{\partial \mathcal{L}_D^n}{\partial \theta_D^n} = \text{nn_backprop}(f, \theta^n, \frac{\partial \mathcal{L}_D^n}{\partial r^n})\\ \theta^{n+1} = \text{update_weights}(\theta^n, \frac{\partial \mathcal{L}_D^n}{\partial \theta_D^n}) \end{array}$

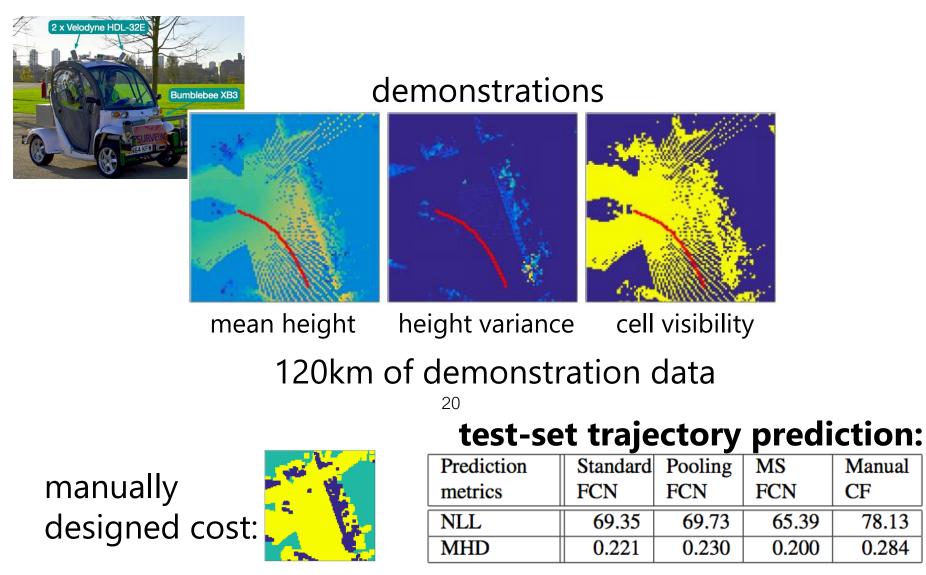
Need to iteratively solve MDP for every weight update

8:

9:

10: end for

Case Study: MaxEnt Deep IRL MaxEnt IRL with known dynamics (tabular setting), neural net cost



MHD: modified Hausdorff distance

slides adapted from C. Finn

Manual

78.13

0.284

CF

Case Study: MaxEnt Deep IRL MaxEnt IRL with known dynamics (tabular setting), neural net cost

Strengths

- scales to neural net costs

Limitations

- still need to repeatedly solve the MDP
- assumes known dynamics

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Break

What about larger RL problems?

- MaxEnt IRL: probabilistic framework for learning reward functions
- Computing gradient requires enumerating state-action visitations for all states and actions
 - Only really viable for small, discrete state and action spaces
 - Amounts to a dynamic programming algorithm (exact forwardbackward inference)
- For deep IRL, we want two things:
 - Large and continuous state and action spaces
 - Effective learning under unknown dynamics

Unknown dynamics & large state/action spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

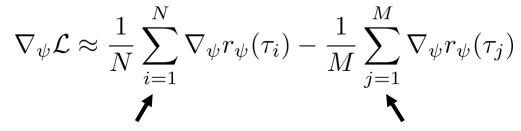
$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$
estimate with expert samples soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm then run this policy to sample $\{\tau_j\}$

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{\psi} r_{\psi}(\tau_j)$$

sum over expert samples sum over policy samples

More efficient sample-based updates



sum over expert samples sum over policy samples

improve learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm (a little) then run this policy to sample $\{\tau_i\}$

looks expensive! what if we use "lazy" policy optimization?

problem: estimator is now biased! wrong distribution! solution: use importance sampling

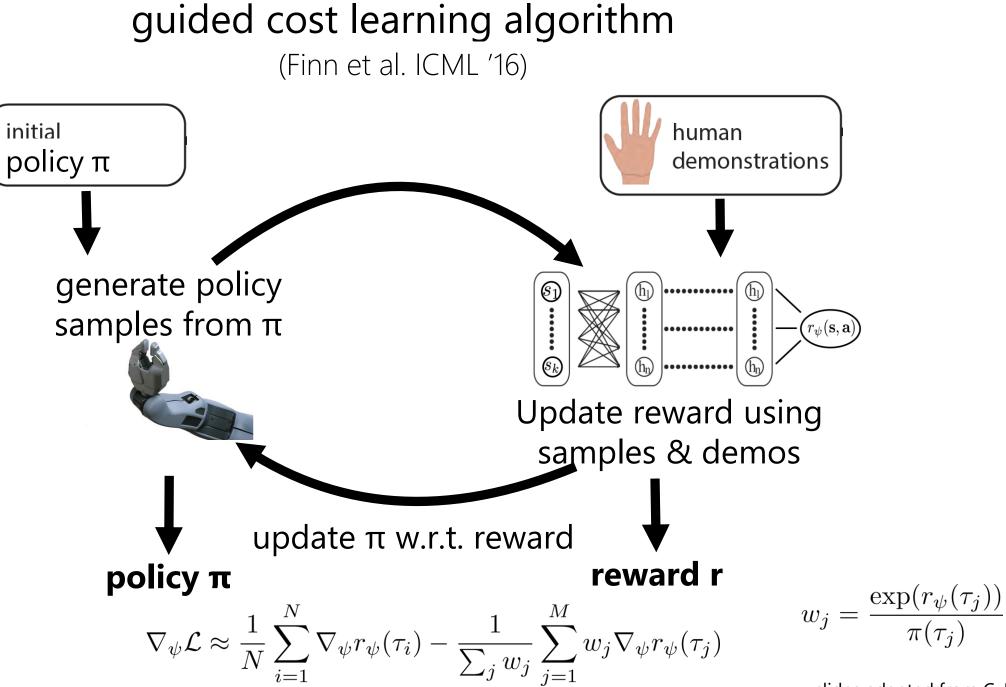
$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{\sum_j w_j} \sum_{j=1}^{M} w_j \nabla_{\psi} r_{\psi}(\tau_j) \qquad w_j = \frac{\exp(r_{\psi}(\tau_j))}{\pi(\tau_j)}$$

Importance sampling

$$\begin{split} \nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{\sum_{j} w_{j}} \sum_{j=1}^{M} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) & w_{j} = \frac{\exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})} \\ & \mathbf{v}_{j} = \frac{\exp(r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\pi(\tau_{j})} \\ & \mathbf{v}_{j} = \frac{\exp(r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\mathbf{v}_{j}(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \pi(\mathbf{a}_{t} | \mathbf{s}_{t})} \\ & \text{which sampling distribution } \pi(\tau) \text{ is best?} & = \frac{\exp(\sum_{t} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}))}{\prod_{t} \pi(\mathbf{a}_{t} | \mathbf{s}_{t})} \\ & \text{optimal IS distribution } q(x) \text{ for } E_{p(x)}[f(x)] \text{ is } q(x) \propto |f(x)| p(x) \\ & \text{ in our case, optimal } \pi \text{ is therefore } \pi(\tau) \propto \exp(r_{\psi}(\tau)) \end{split}$$

max-ent optimal policy for r_ψ

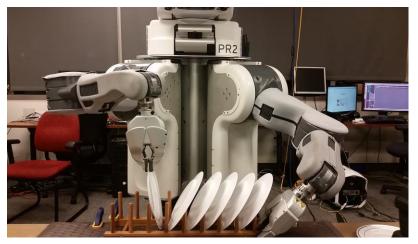
each policy update w.r.t. r_{ψ} brings us closer to the optimal distribution!



Guided Cost Learning Experiments

Real-world Tasks

dish placement



state includes goal plate pose

pouring almonds

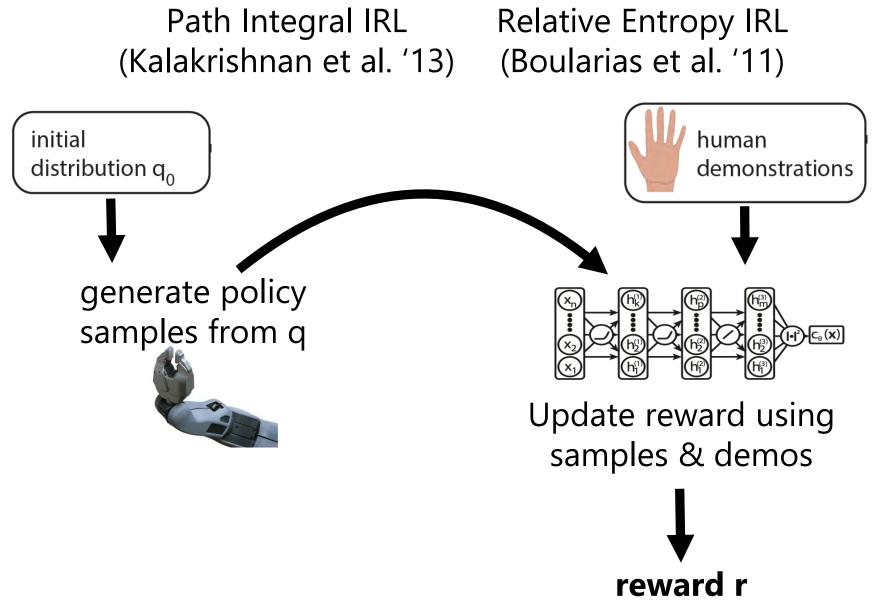


state includes unsupervised visual features [Finn et al. '16]

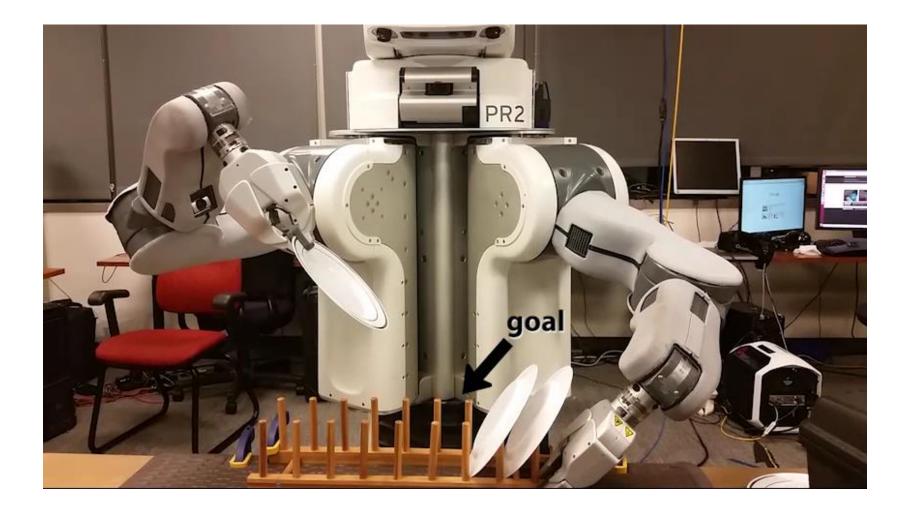
Comparison

Relative Entropy IRL (Boularias et al. '11)

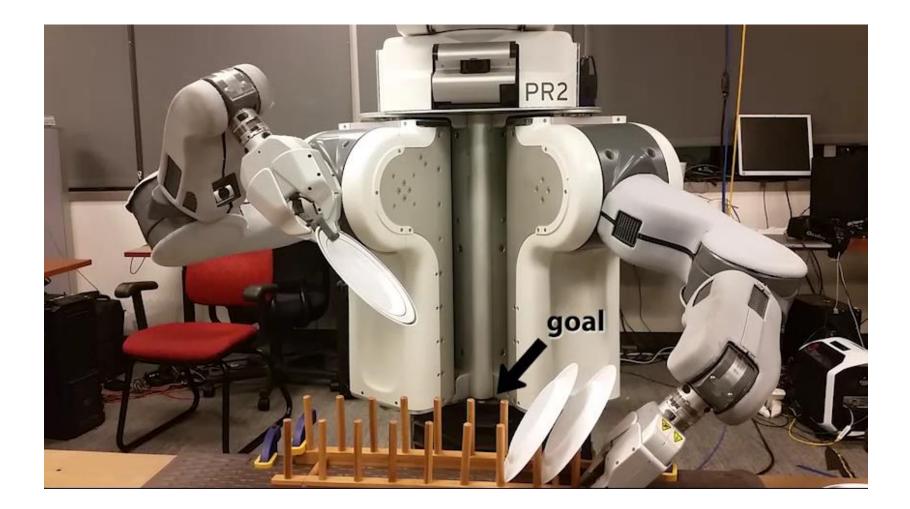
Comparisons



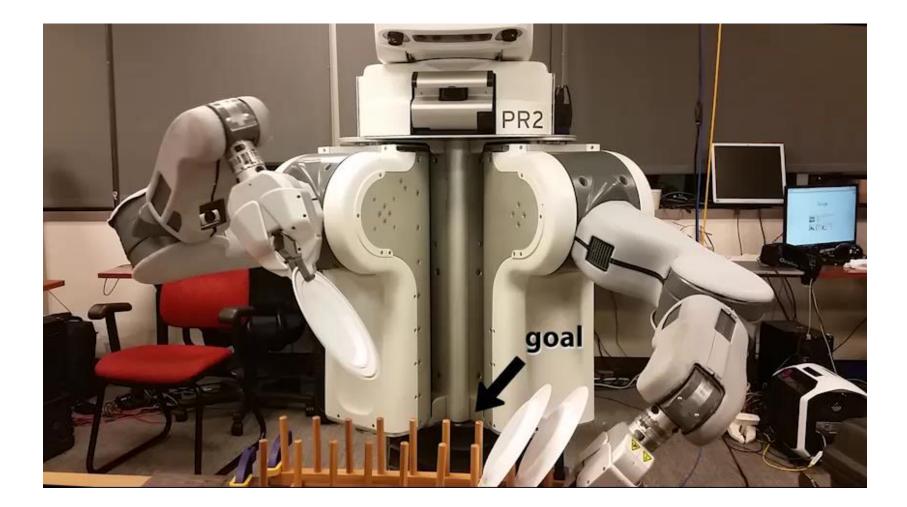
Dish placement, standard reward



Dish placement, RelEnt IRL



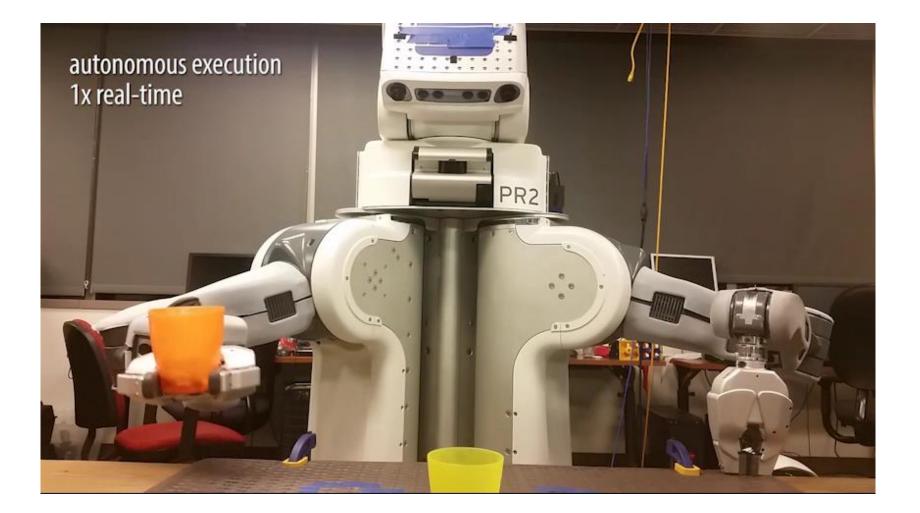
Dish placement, GCL policy



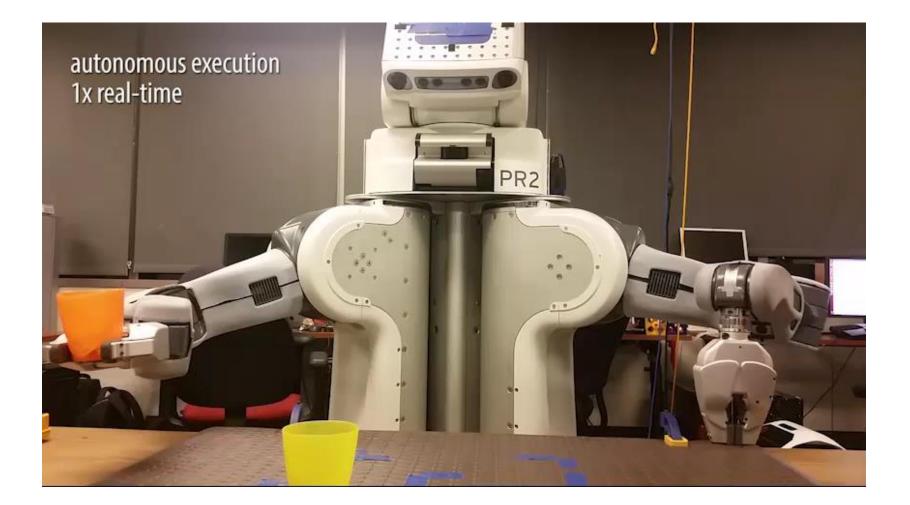
Pouring, demos



Pouring, RelEnt IRL



Pouring, GCL policy



Aside: Generative Adversarial Networks

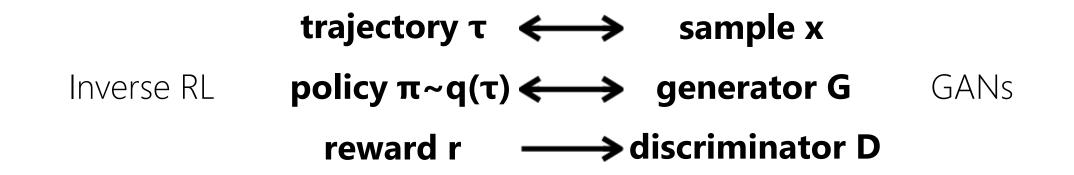
Arjovsky et al. '17

(Goodfellow et al. '14)

Similar to inverse RL, GANs learn an objective for generative modeling.



Zhu et al. '17



(Finn*, Christiano*, et al. '16)

OUTPUT

Isola et al. '17

Connection to Generative Adversarial Networks

trajectory $\tau \leftrightarrow \rightarrow$ sample x policy $\pi \sim q(\tau) \iff$ generator G Inverse RL GANS reward r — discriminator D data distribution p Reward/discriminator optimization: $D^*(\tau) = \frac{p(\tau)}{p(\tau) + q(\tau)}$ $D_{\psi}(\tau) = \frac{\frac{1}{Z} \exp(R_{\psi})}{\frac{1}{Z} \exp(R_{\psi}) + q(\tau)}$ $\mathcal{L}_{\text{discriminator}}(\psi) = \mathbb{E}_{\tau \sim p} \left[-\log D_{\psi}(\tau) \right] + \mathbb{E}_{\tau \sim q} \left[-\log(1 - D_{\psi}(\tau)) \right]$

(Finn*, Christiano*, et al. '16)

Connection to Generative Adversarial Networks

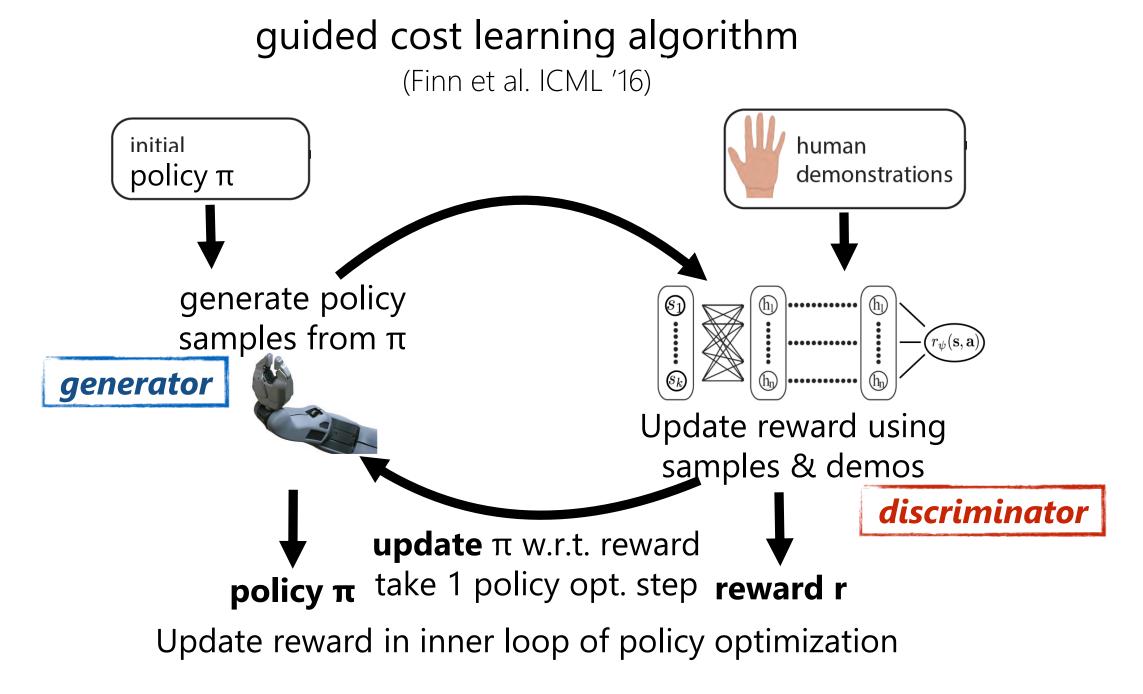
trajectory τsample xInverse RLpolicy π~q(τ)generator GGANsreward rdiscriminator Ddata distribution p

Policy/generator optimization: $\mathcal{L}_{generator}(\theta) = \mathbb{E}_{\tau \sim q} [\log(1 - D_{\psi}(\tau)) - \log D_{\psi}(\tau)]$ $= \mathbb{E}_{\tau \sim q} [\log q(\tau) + \log Z - R_{\psi}(\tau)]$

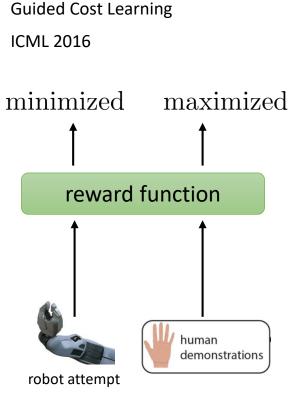
entropy-regularized RL

Unknown dynamics: train generator/policy with RL

Baram et al. ICML '17: use learned dynamics model to backdrop through discriminator (Finn*, Christiano*, et al. '16)



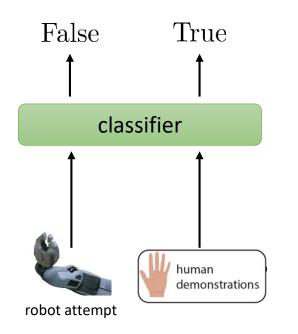
IRL as adversarial optimization



learns distribution $p(\tau)$ such that demos have max likelihood $p(\tau) \propto \exp(r(\tau))$ (MaxEnt model)



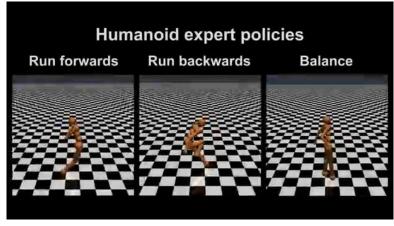
Generative Adversarial Imitation Learning Ho & Ermon, NIPS 2016



 $D(\tau) =$ probability τ is a demo

use $\log D(\tau)$ as "reward"

same thing!



Hausman, Chebotar, Schaal, Sukhatme, Lim



Merel, Tassa, TB, Srinivasan, Lemmon, Wang, Wayne, Heess

Generative Adversarial Imitation Learning Experiments (Ho & Ermon NIPS '16)

learned behaviors from human motion capture Merel et al. '17

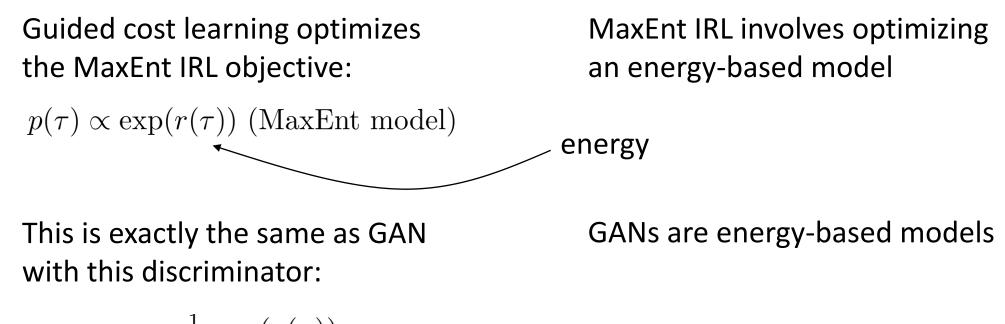


walking



falling & getting up

IRL = generative adversarial networks = energy-based models



$$D(\tau) = \frac{\frac{1}{Z} \exp(r(\tau))}{\frac{1}{Z} \exp(r(\tau)) + \pi(\tau)}$$

Finn*, Christiano*, Abbeel, L. '16

Review

- IRL: infer unknown reward from expert demonstrations
- MaxEnt IRL: infer reward by learning under the control-as-inference framework
- MaxEnt IRL with dynamic programming: simple and efficient, but requires small state space and known dynamics
- Differential MaxEnt IRL: good for large, continuous spaces, but requires known dynamics and is local
- Sampling-based MaxEnt IRL: generate samples to estimate the partition function
 - Guided cost learning algorithm
 - Connection to generative adversarial networks
 - Generative adversarial imitation learning (not IRL per se, but very similar)

Suggested Reading on Inverse RL

Classic Papers:

Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:

Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks