Connections Between Inference and Control

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

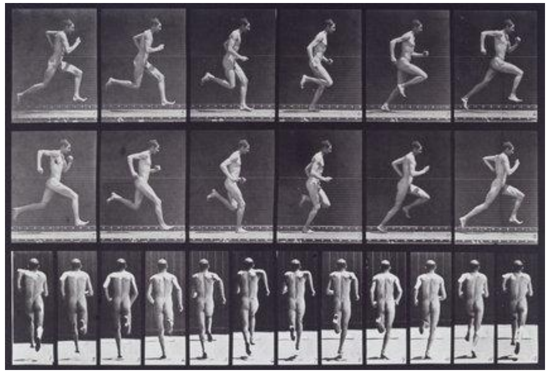
1. Homework 3 is due today, at 11:59 pm
2. Homework 4 comes out tonight
3. Final project proposal due on Monday!

Today’s Lecture

1. Does reinforcement learning and optimal control provide a reasonable model of human behavior?
2. Is there a better explanation?
3. Can we derive optimal control, reinforcement learning, and planning as *probabilistic inference*?
4. How does this change our RL algorithms?
5. (next week) We’ll see this is crucial for *inverse* reinforcement learning

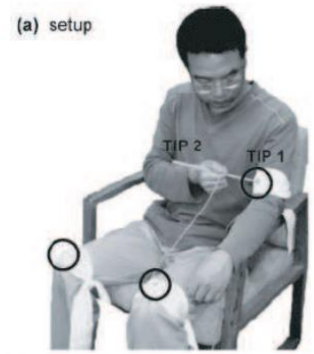
- Goals:
    - Understand the connection between inference and control
    - Understand how specific RL algorithms can be instantiated in this framework
    - Understand why this might be a good idea

Optimal Control as a Model of Human Behavior



Muybridge (c. 1870)

Mombaur et al. ‘09



Li & Todorov ‘06

Ziebart ‘08

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

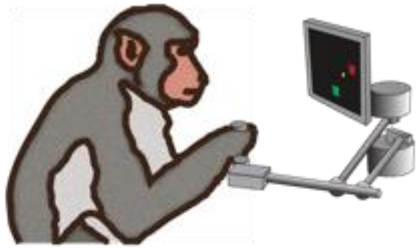
$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

optimize this to explain the data

$$\pi = \arg \max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$$

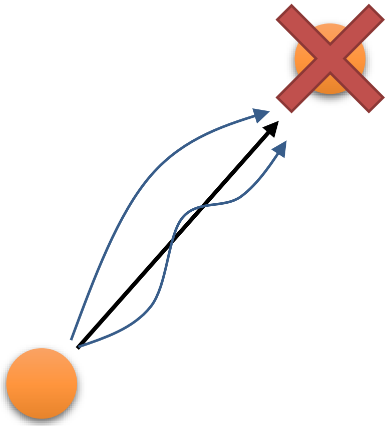
What if the data is **not** optimal?



some mistakes matter more than others!

behavior is **stochastic**

but good behavior is still the most likely



A probabilistic graphical model of decision making

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

$$\underbrace{p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T})}_{\tau} = ??$$

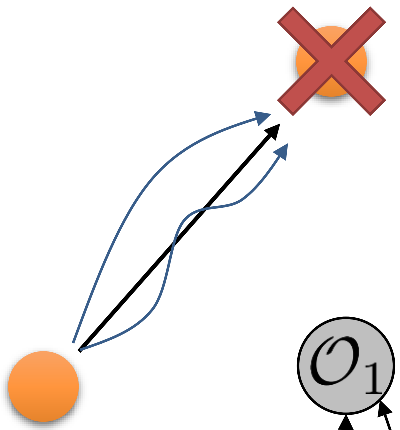
no assumption of optimal behavior!

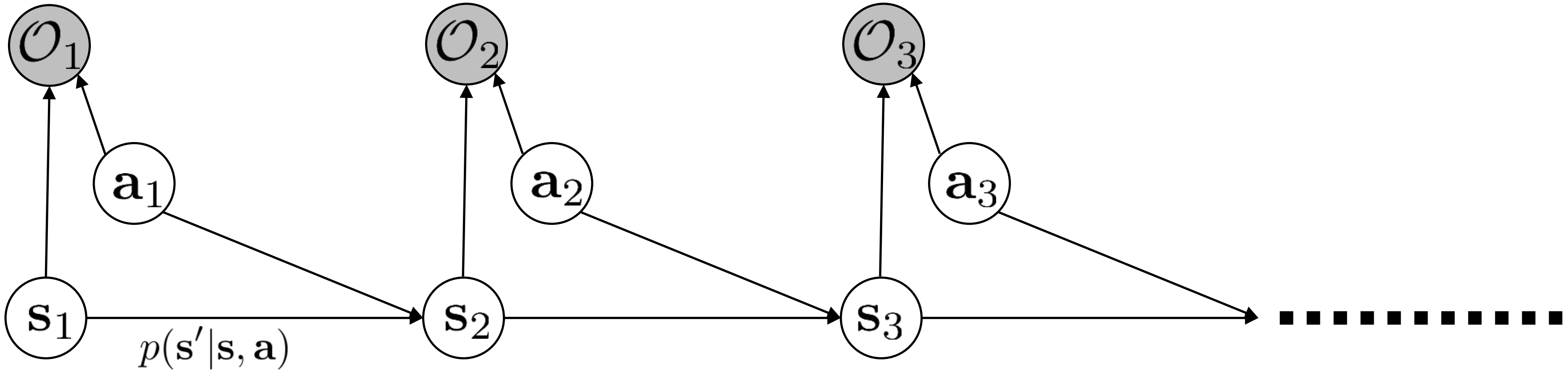
$$p(\tau | \mathcal{O}_{1:T})$$

$$p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

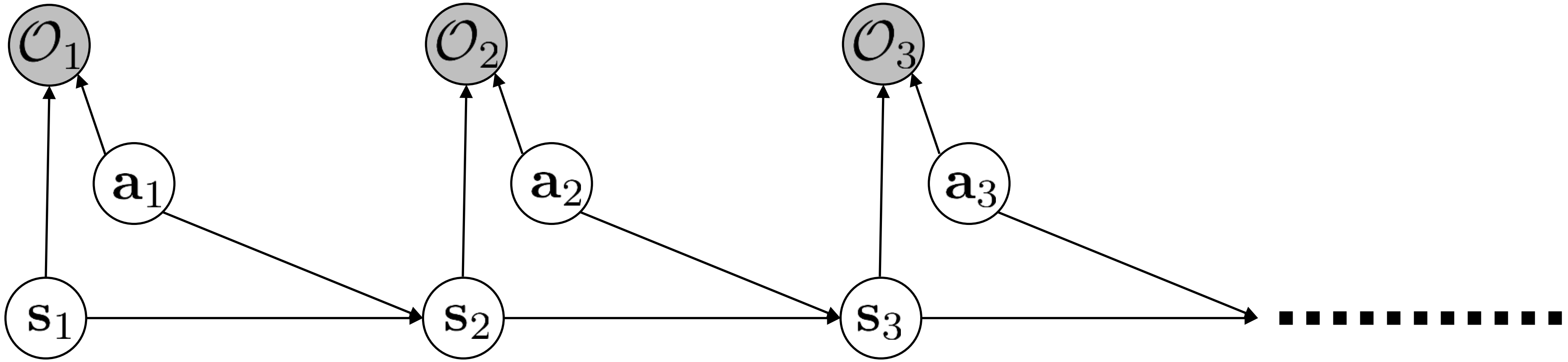
$$p(\tau | \mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$

$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$



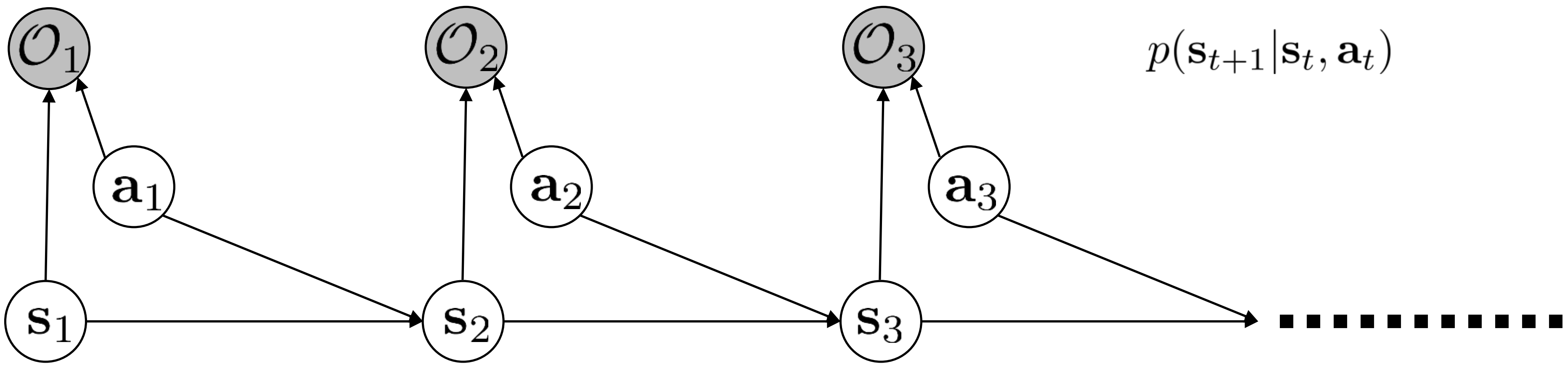


Why is this interesting?



- Can model suboptimal behavior (important for inverse RL)
- Can apply inference algorithms to solve control and planning problems
- Provides an explanation for why stochastic behavior might be preferred (useful for exploration and transfer learning)

Inference = planning



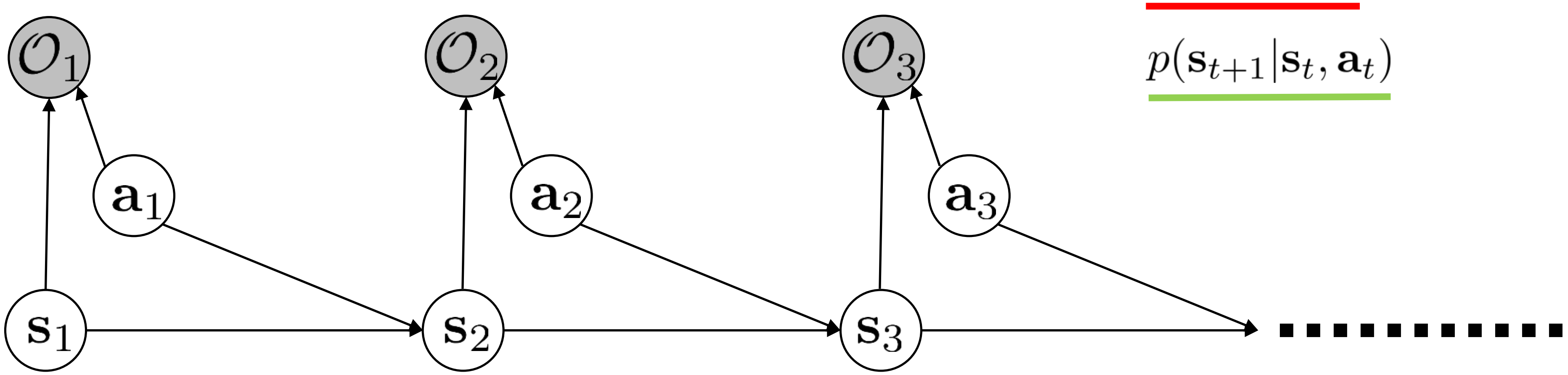
$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

how to do inference?

1. compute backward messages $\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t, \mathbf{a}_t)$
2. compute policy $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T})$
3. compute forward messages $\alpha_t(\mathbf{s}_t) = p(\mathbf{s}_t|\mathcal{O}_{1:t-1})$

Backward messages



$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t, \mathbf{a}_t)$$
$$= \int p(\mathcal{O}_{t:T}, \mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_{t+1}$$
$$= \int p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_{t+1}$$

$$p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}) = \int p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) p(\mathbf{a}_{t+1}|\mathbf{s}_{t+1}) d\mathbf{a}_{t+1}$$

$\beta_t(\mathbf{s}_{t+1}, \mathbf{a}_{t+1})$

which actions are likely *a priori*
(assume uniform for now)

for $t = T-1$ to 1:

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[\beta_{t+1}(\mathbf{s}_{t+1})]$$
$$\beta_t(\mathbf{s}_t) = E_{\mathbf{a}_t \sim p(\mathbf{a}_t|\mathbf{s}_t)}[\beta_t(\mathbf{s}_t, \mathbf{a}_t)]$$

A closer look at the backward pass

for $t = T-1$ to 1:

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[\beta_{t+1}(\mathbf{s}_{t+1})]$$

$$\beta_t(\mathbf{s}_t) = E_{\mathbf{a}_t \sim p(\mathbf{a}_t|\mathbf{s}_t)}[\beta_t(\mathbf{s}_t, \mathbf{a}_t)]$$

value iteration algorithm:

1. set $Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E[V(\mathbf{s}')]$
2. set $V(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a})$

let $V_t(\mathbf{s}_t) = \log \beta_t(\mathbf{s}_t)$

let $Q_t(\mathbf{s}_t, \mathbf{a}_t) = \log \beta_t(\mathbf{s}_t, \mathbf{a}_t)$

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t)) d\mathbf{a}_t$$

$V_t(\mathbf{s}_t) \rightarrow \max_{\mathbf{a}_t} Q_t(\mathbf{s}_t, \mathbf{a}_t)$ as $Q_t(\mathbf{s}_t, \mathbf{a}_t)$ gets bigger!

"optimistic" transition (not a good idea!)

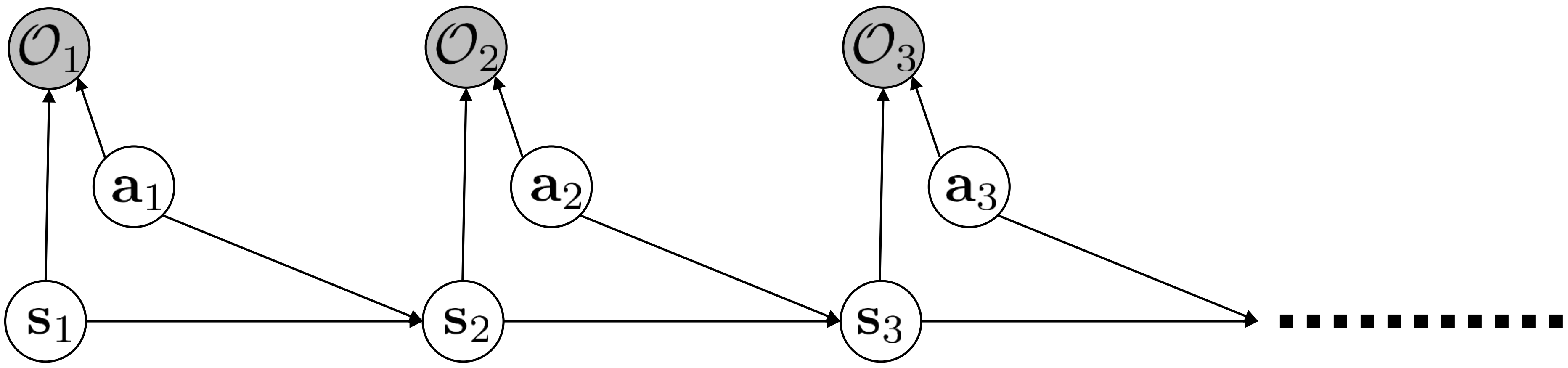
$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \log E[\exp(V_{t+1}(\mathbf{s}_{t+1}))]$$

deterministic transition: $Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + V_{t+1}(\mathbf{s}_{t+1})$

a better stochastic model: $Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E[V_{t+1}(\mathbf{s}_{t+1})]$

Ziebart et al. '10 "Modeling Interaction via the Principle of Maximum Causal Entropy"

Backward pass summary



$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t, \mathbf{a}_t)$

probability that we can be optimal at steps t through T given that we take action $\mathbf{a}_t$ in state $\mathbf{s}_t$

for $t = T - 1$ to 1:

$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[\beta_{t+1}(\mathbf{s}_{t+1})]$

compute recursively from $t = T$ to $t = 1$

$\beta_t(\mathbf{s}_t) = E_{\mathbf{a}_t \sim p(\mathbf{a}_t|\mathbf{s}_t)}[\beta_t(\mathbf{s}_t, \mathbf{a}_t)]$

let $V_t(\mathbf{s}_t) = \log \beta_t(\mathbf{s}_t)$

let $Q_t(\mathbf{s}_t, \mathbf{a}_t) = \log \beta_t(\mathbf{s}_t, \mathbf{a}_t)$

log of β_t is "Q-function-like"

The action prior

remember this?

$$p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}) = \int p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) p(\mathbf{a}_{t+1}|\mathbf{s}_{t+1}) d\mathbf{a}_{t+1}$$

$\beta_t(\mathbf{s}_{t+1}, \mathbf{a}_{t+1})$

("soft max")

what if the action prior is not uniform?

$$V(\mathbf{s}_t) = \log \int \exp(Q(\mathbf{s}_t, \mathbf{a}_t) + \log p(\mathbf{a}_t|\mathbf{s}_t))\mathbf{a}_t$$

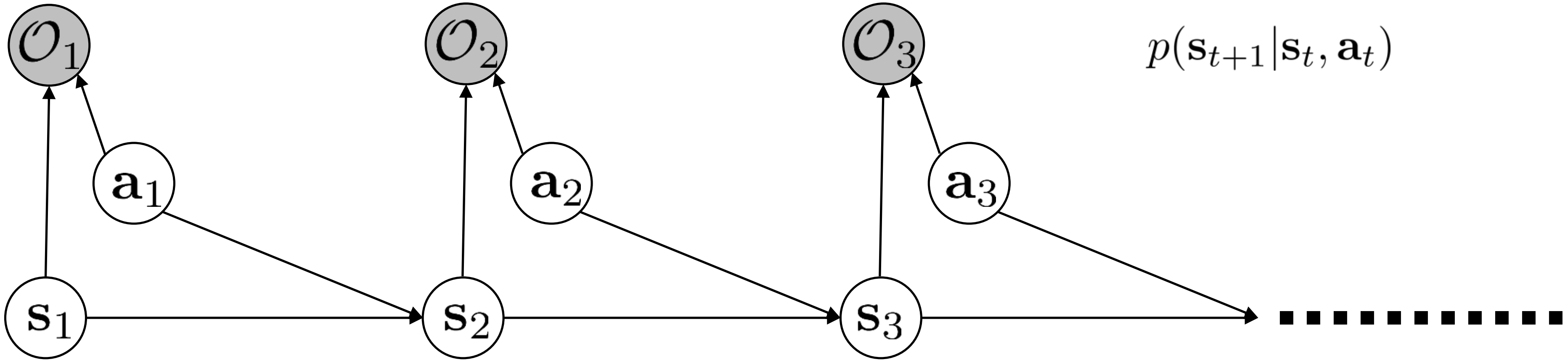
$$Q(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E[V(\mathbf{s}_{t+1})]$$

let $\tilde{Q}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \log p(\mathbf{a}_t|\mathbf{s}_t) + E[V(\mathbf{s}_{t+1})]$

$$V(\mathbf{s}_t) = \log \int \exp(\tilde{Q}(\mathbf{s}_t, \mathbf{a}_t))\mathbf{a}_t \quad \Leftrightarrow \quad V(\mathbf{s}_t) = \log \int \exp(Q(\mathbf{s}_t, \mathbf{a}_t) + \log p(\mathbf{a}_t|\mathbf{s}_t))\mathbf{a}_t$$

can **always** fold the action prior into the reward! uniform action prior can be assumed without loss of generality

Policy computation



$$p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t) \propto \exp(r(\mathbf{s}_t,\mathbf{a}_t))$$

$$p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)$$

2. compute policy $p(\mathbf{a}_t|\mathbf{s}_t,\mathcal{O}_{1:T})$

$$\beta_t(\mathbf{s}_t,\mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t,\mathbf{a}_t)$$

$$\beta_t(\mathbf{s}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t)$$

$$\begin{aligned}
p(\mathbf{a}_t|\mathbf{s}_t,\mathcal{O}_{1:T}) &= \pi(\mathbf{a}_t|\mathbf{s}_t) \\
&= p(\mathbf{a}_t|\mathbf{s}_t,\mathcal{O}_{t:T}) \\
&= \frac{p(\mathbf{a}_t,\mathbf{s}_t|\mathcal{O}_{t:T})}{p(\mathbf{s}_t|\mathcal{O}_{t:T})} \\
&= \frac{p(\mathcal{O}_{t:T}|\mathbf{a}_t,\mathbf{s}_t)p(\mathbf{a}_t,\mathbf{s}_t)/\cancel{p(\mathcal{O}_{t:T})}}{p(\mathcal{O}_{t:T}|\mathbf{s}_t)p(\mathbf{s}_t)/\cancel{p(\mathcal{O}_{t:T})}} \\
&= \frac{p(\mathcal{O}_{t:T}|\mathbf{a}_t,\mathbf{s}_t)}{p(\mathcal{O}_{t:T}|\mathbf{s}_t)}\frac{p(\mathbf{a}_t,\mathbf{s}_t)}{p(\mathbf{s}_t)} = \frac{\beta_t(\mathbf{s}_t,\mathbf{a}_t)}{\beta_t(\mathbf{s}_t)}\cancel{p(\mathbf{a}_t|\mathbf{s}_t)}
\end{aligned}$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \frac{\beta_t(\mathbf{s}_t,\mathbf{a}_t)}{\beta_t(\mathbf{s}_t)}$$

Policy computation with value functions

for $t = T-1$ to 1:

$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E[V_{t+1}(\mathbf{s}_{t+1})]$$

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t))\mathbf{a}_t$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \frac{\beta_t(\mathbf{s}_t, \mathbf{a}_t)}{\beta_t(\mathbf{s}_t)}$$

$$V_t(\mathbf{s}_t) = \log \beta_t(\mathbf{s}_t)$$

$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = \log \beta_t(\mathbf{s}_t, \mathbf{a}_t)$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t, \mathbf{a}_t))$$

variants:

discounted SOC: $Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma E[V_{t+1}(\mathbf{s}_{t+1})]$

explicit temperature: $V_t(\mathbf{s}_t) = \alpha \log \int \exp\left(\frac{1}{\alpha} Q_t(\mathbf{s}_t, \mathbf{a}_t)\right) d\mathbf{a}_t$

Policy computation summary

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t, \mathbf{a}_t))$$

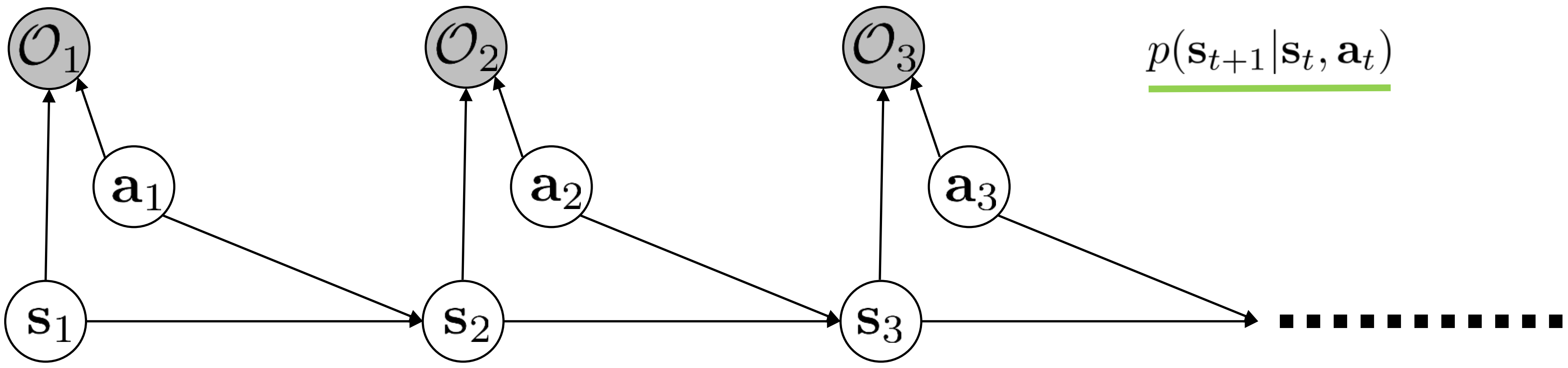
$$\text{with temperature: } \pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(\tfrac{1}{\alpha}Q_t(\mathbf{s}_t, \mathbf{a}_t) - \tfrac{1}{\alpha}V_t(\mathbf{s}_t)) = \exp(\tfrac{1}{\alpha}A_t(\mathbf{s}_t, \mathbf{a}_t))$$

- Natural interpretation: better actions are more probable
- Random tie-breaking
- Analogous to Boltzmann exploration
- Approaches greedy policy as temperature decreases

Forward messages

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$



$$\alpha_t(\mathbf{s}_t) = p(\mathbf{s}_t|\mathcal{O}_{1:t-1})$$

$$= \int \underbrace{p(\mathbf{s}_t|\mathbf{s}_{t-1}, \mathbf{a}_{t-1})}p(\mathbf{a}_{t-1}|\mathbf{s}_{t-1}, \mathcal{O}_{t-1})\underbrace{p(\mathbf{s}_{t-1}|\mathcal{O}_{1:t-2})}_{\alpha_{t-1}(\mathbf{s}_{t-1})}d\mathbf{s}_{t-1}d\mathbf{a}_{t-1}$$

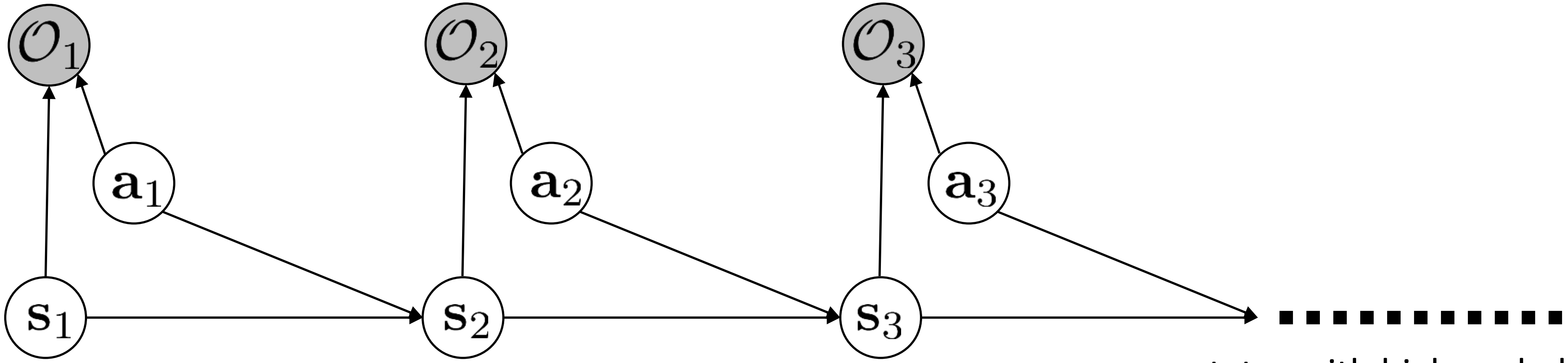
$\alpha_1(\mathbf{s}_1) = p(\mathbf{s}_1)$ (usually known)

$$p(\mathbf{a}_{t-1}|\mathbf{s}_{t-1}, \mathcal{O}_{t-1}) = \frac{p(\mathcal{O}_{t-1}|\mathbf{s}_{t-1}, \mathbf{a}_{t-1})p(\mathbf{a}_{t-1}|\mathbf{s}_{t-1})}{p(\mathcal{O}_{t-1}|\mathbf{s}_{t-1})}$$

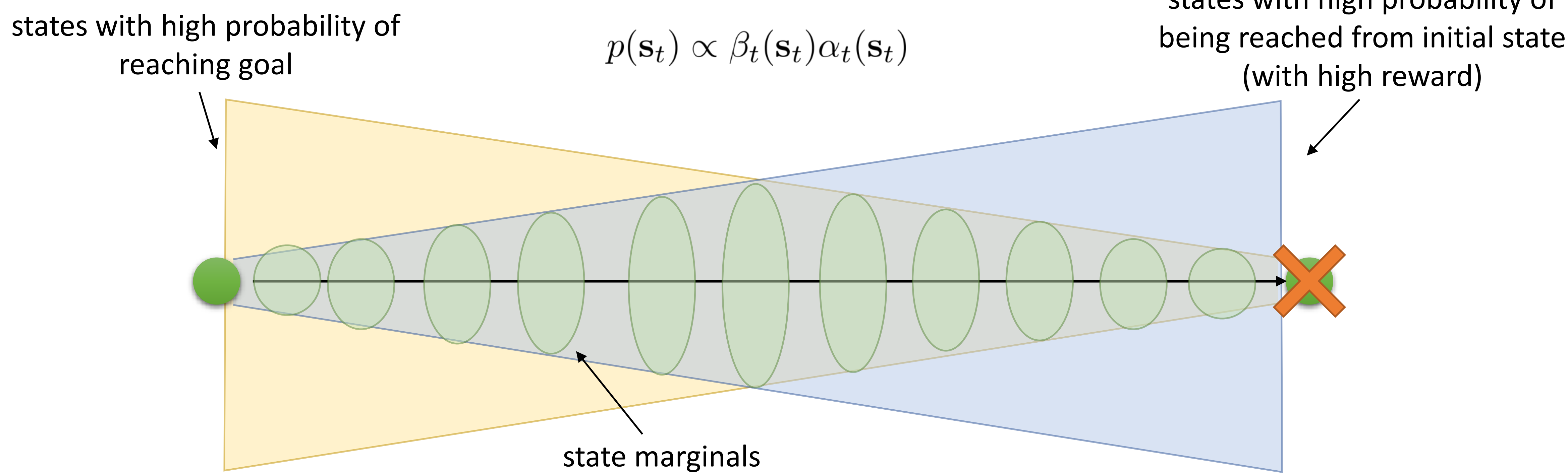
what if we want $p(\mathbf{s}_t|\mathcal{O}_{1:T})$?

$$p(\mathbf{s}_t|\mathcal{O}_{1:T}) = \frac{p(\mathbf{s}_t, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})} = \frac{\overbrace{p(\mathcal{O}_{t:T}|\mathbf{s}_t)}^{\beta_t(\mathbf{s}_t)}p(\mathbf{s}_t, \mathcal{O}_{1:t-1})}{p(\mathcal{O}_{1:T})} \propto \beta_t(\mathbf{s}_t)\underbrace{p(\mathbf{s}_t|\mathcal{O}_{1:t-1})}_{\alpha_t(\mathbf{s}_t)}\cancel{p(\mathcal{O}_{1:t-1})} \propto \beta_t(\mathbf{s}_t)\alpha_t(\mathbf{s}_t)$$

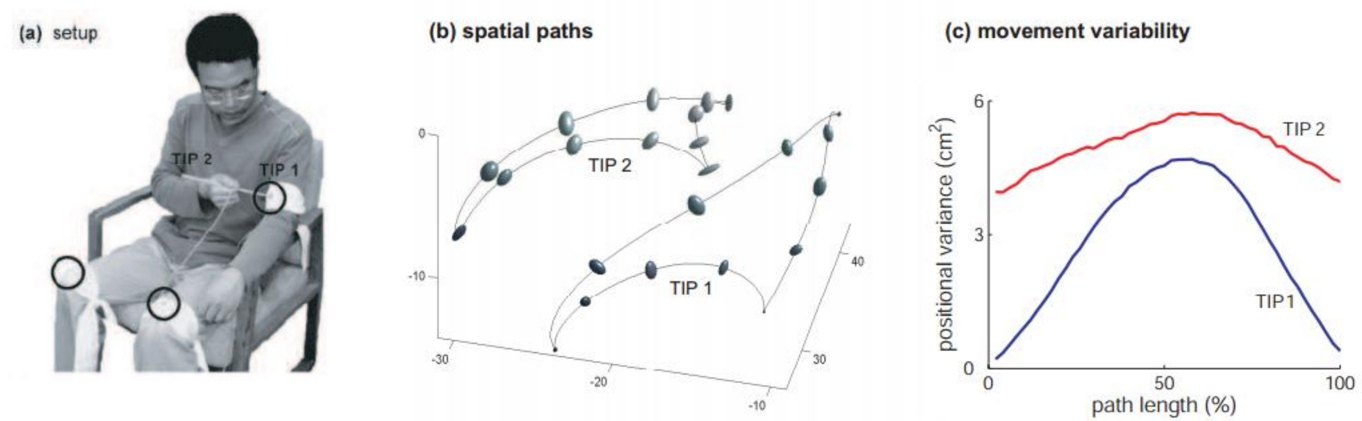
Forward/backward message intersection



$$p(\mathbf{s}_t) \propto \beta_t(\mathbf{s}_t)\alpha_t(\mathbf{s}_t)$$



Forward/backward message intersection

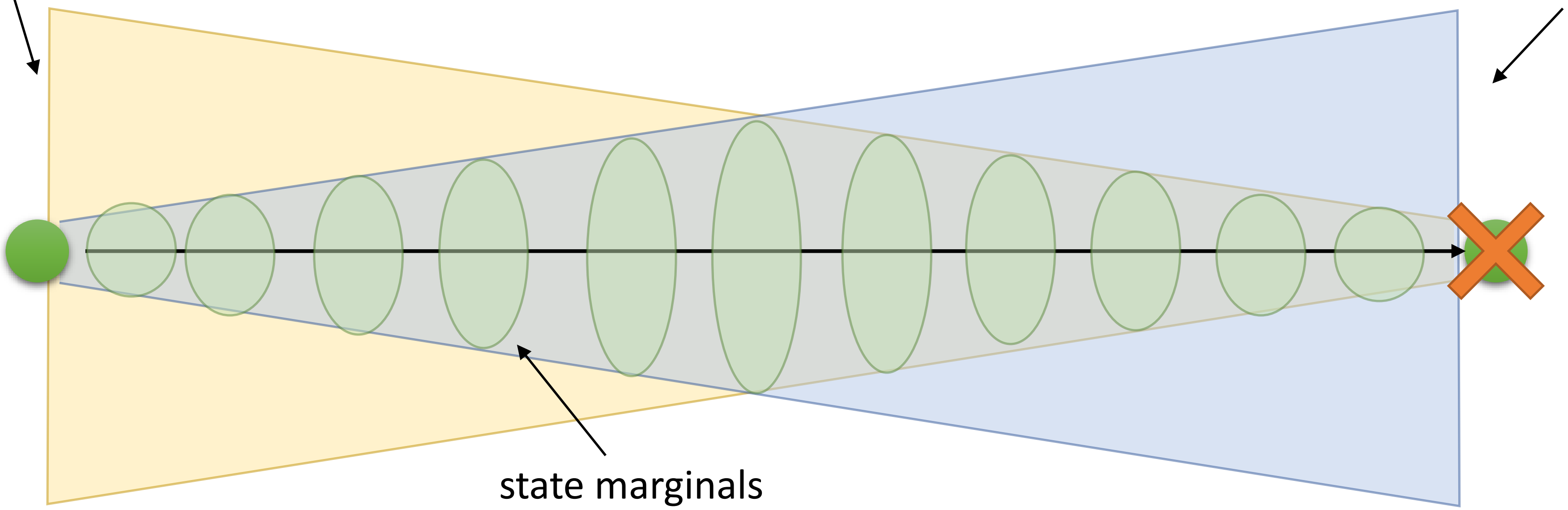


Li & Todorov, 2006

states with high probability of reaching goal

$$p(\mathbf{s}_t) \propto \beta_t(\mathbf{s}_t)\alpha_t(\mathbf{s}_t)$$

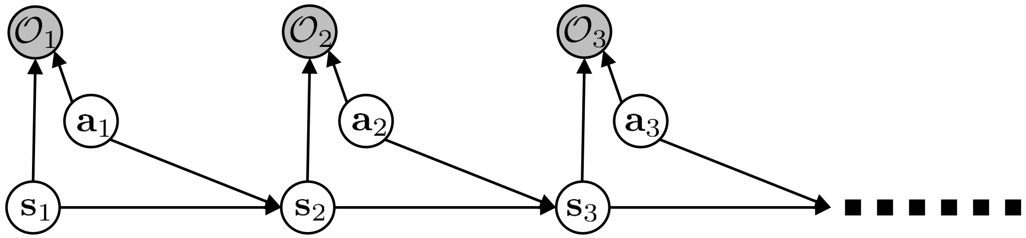
states with high probability of being reached from initial state (with high reward)



state marginals

Summary

1. Probabilistic graphical model for optimal control



2. Control = inference (similar to HMM, EKF, etc.)

3. Very similar to dynamic programming, value iteration, etc. (but “soft”)

Q-learning with soft optimality

standard Q-learning: $\phi \leftarrow \phi + \alpha \nabla_\phi Q_\phi(\mathbf{s}, \mathbf{a})(r(\mathbf{s}, \mathbf{a}) + \gamma V(\mathbf{s}') - Q_\phi(\mathbf{s}, \mathbf{a}))$

target value: $V(\mathbf{s}') = \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')$

soft Q-learning: $\phi \leftarrow \phi + \alpha \nabla_\phi Q_\phi(\mathbf{s}, \mathbf{a})(r(\mathbf{s}, \mathbf{a}) + \gamma V(\mathbf{s}') - Q_\phi(\mathbf{s}, \mathbf{a}))$

target value: $V(\mathbf{s}') = \text{soft}\max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}') = \log \int \exp(Q_\phi(\mathbf{s}', \mathbf{a}')) d\mathbf{a}'$

$\pi(\mathbf{a}|\mathbf{s}) = \exp(Q_\phi(\mathbf{s}, \mathbf{a}) - V(\mathbf{s})) = \exp(A(\mathbf{s}, \mathbf{a}))$

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{R}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{R}$ uniformly
3. compute $y_j = r_j + \gamma \text{soft}\max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. update ϕ': copy ϕ every N steps, or Polyak average $\phi' \leftarrow \tau\phi' + (1-\tau)\phi$

Policy gradient with soft optimality

$$\pi(\mathbf{a}|\mathbf{s}) = \exp(Q_\phi(\mathbf{s},\mathbf{a}) - V(\mathbf{s})) \text{ optimizes } \sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t)] + E_{\pi(\mathbf{s}_t)}[\mathcal{H}(\pi(\mathbf{a}_t|\mathbf{s}_t))]$$

policy entropy

intuition: $\pi(\mathbf{a}|\mathbf{s}) \propto \exp(Q_\phi(\mathbf{s},\mathbf{a}))$ when π minimizes $D_{\mathrm{KL}}(\pi(\mathbf{a}|\mathbf{s}) \| \frac{1}{Z}\exp(Q(\mathbf{s},\mathbf{a})))$

$$D_{\mathrm{KL}}(\pi(\mathbf{a}|\mathbf{s}) \| \tfrac{1}{Z}\exp(Q(\mathbf{s},\mathbf{a}))) = E_{\pi(\mathbf{a}|\mathbf{s})}[Q(\mathbf{s},\mathbf{a})] - \mathcal{H}(\pi)$$

often referred to as “entropy regularized” policy gradient

combats premature entropy collapse

turns out to be closely related to soft Q-learning:
see Haarnoja et al. ‘17 and Schulman et al. ‘17

Ziebart et al. ‘10 “Modeling Interaction via the Principle of Maximum Causal Entropy”

Policy gradient vs Q-learning

policy gradient derivation:

$$J(\theta) = \sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t)] + E_{\pi(\mathbf{s}_t)}[\underbrace{\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_t))}_{E_{\pi(\mathbf{a}_t|\mathbf{s}_t)}[-\log\pi(\mathbf{a}_t|\mathbf{s}_t)]}] = \sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t) - \log\pi(\mathbf{a}_t|\mathbf{s}_t)]$$

$$\nabla_\theta\left[\sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t) - \log\pi(\mathbf{a}_t|\mathbf{s}_t)]\right]$$

$$\approx \frac{1}{N}\sum_i\sum_t \nabla_\theta\log\pi(\mathbf{a}_t|\mathbf{s}_t)\left(r(\mathbf{s}_t,\mathbf{a}_t) + \underbrace{\left(\sum_{t'=t+1}^{T} r(\mathbf{s}_{t'},\mathbf{a}_{t'}) - \log\pi(\mathbf{a}_{t'}|\mathbf{s}_{t'})\right)}_{\approx Q(\mathbf{s}_{t+1},\mathbf{a}_{t+1})} - \log\pi(\mathbf{a}_t|\mathbf{s}_t) - 1\right)$$

can ignore (baseline)

recall: $\log\pi(\mathbf{a}_t|\mathbf{s}_t) = Q(\mathbf{s}_t,\mathbf{a}_t) - V(\mathbf{s}_t)$

$$\approx \frac{1}{N}\sum_i\sum_t (\nabla_\theta Q(\mathbf{a}_t|\mathbf{s}_t) - \nabla_\theta V(\mathbf{s}_t))\left(r(\mathbf{s}_t,\mathbf{a}_t) + Q(\mathbf{s}_{t+1},\mathbf{a}_{t+1}) - Q(\mathbf{s}_t,\mathbf{a}_t) + V(\mathbf{s}_t)\right)$$

Q-learning: $$-\frac{1}{N}\sum_i\sum_t \nabla_\theta Q(\mathbf{a}_t|\mathbf{s}_t)\left(r(\mathbf{s}_t,\mathbf{a}_t) + \operatorname{soft}\max_{\mathbf{a}_{t+1}} Q(\mathbf{s}_{t+1},\mathbf{a}_{t+1}) - Q(\mathbf{s}_t,\mathbf{a}_t)\right)$$

descent (vs ascent)

off-policy correction

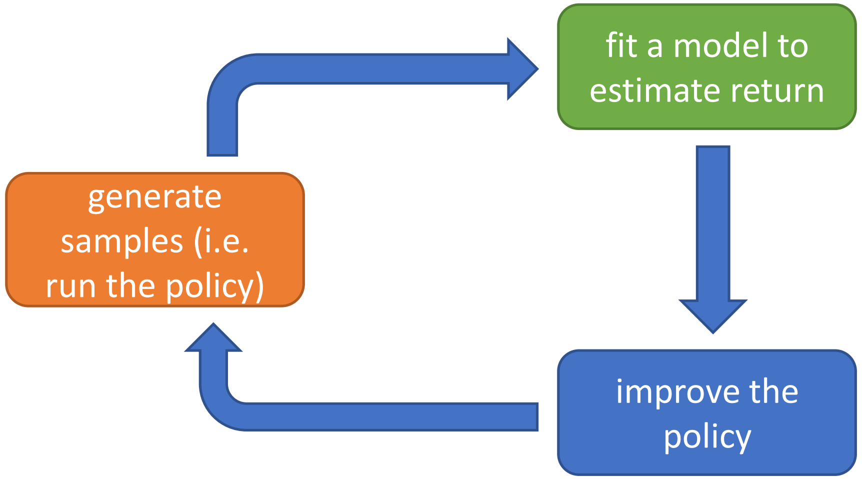
Benefits of soft optimality

- Improve exploration and prevent entropy collapse
- Easier to specialize (finetune) policies for more specific tasks
- Principled approach to break ties
- Better robustness (due to wider coverage of states)
- Can reduce to hard optimality as reward magnitude increases
- Good model for modeling human behavior (more on this later)

Review

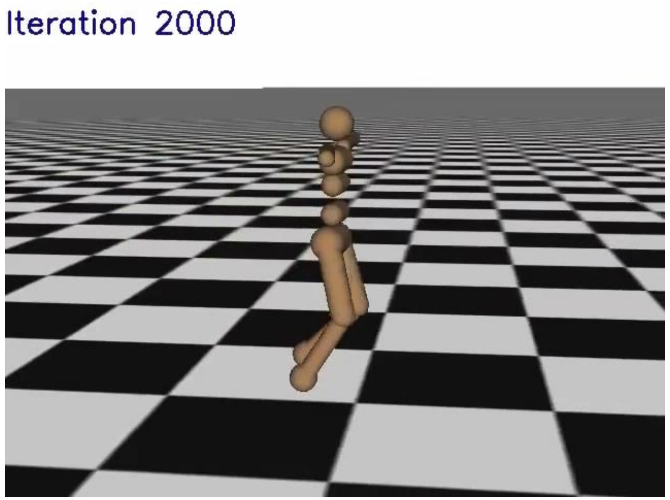
- Reinforcement learning can be viewed as inference in a graphical model
  - Value function is a backward message
  - Maximize reward and entropy (the bigger the rewards, the less entropy matters)
- Soft Q-learning
- Entropy-regularized policy gradient

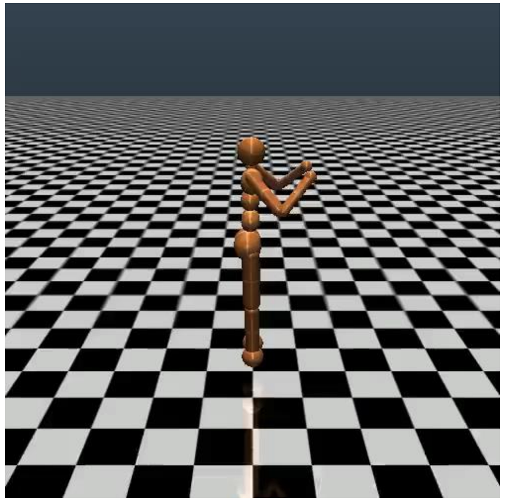
$$Q_\phi(\mathbf{s},\mathbf{a}) \leftarrow r(\mathbf{s},\mathbf{a}) + \gamma \text{soft max}_{\mathbf{a}'} Q_\phi(\mathbf{s}',\mathbf{a}')$$

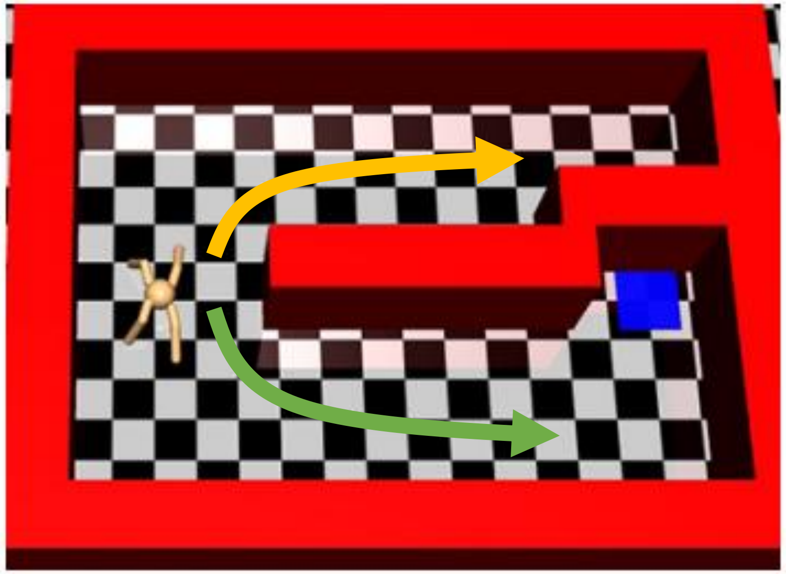


$$\mathbf{a} = \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s},\mathbf{a})$$

Stochastic models for learning control



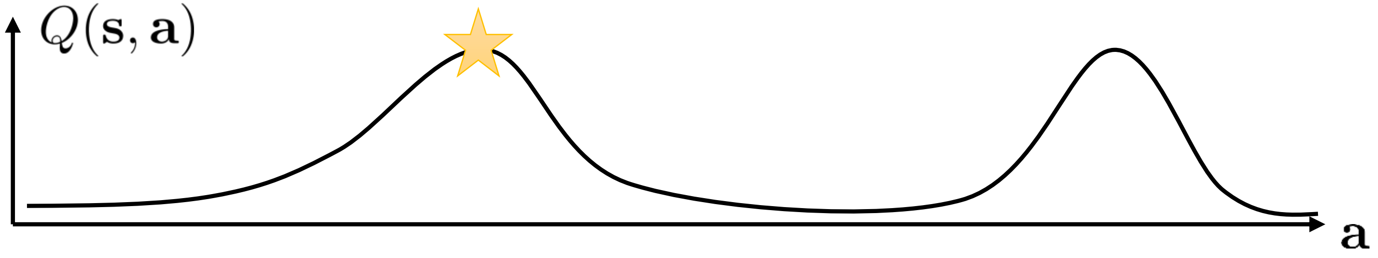


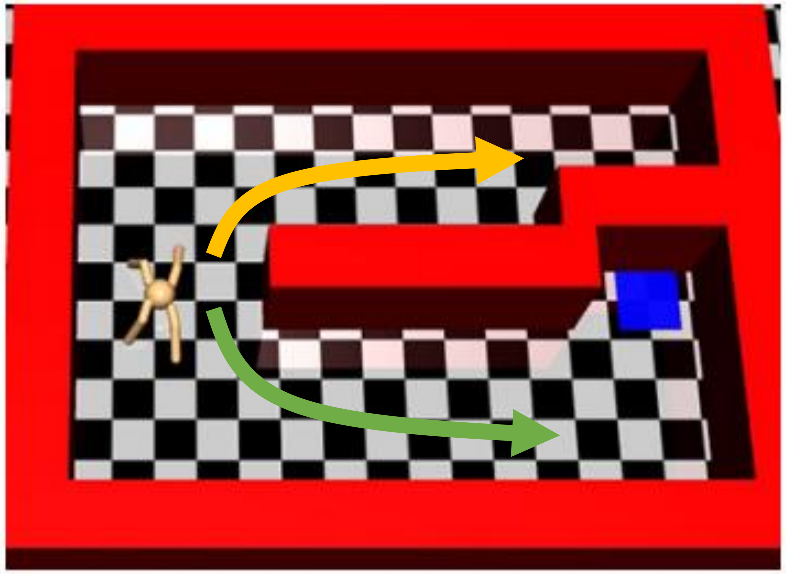


- How can we track *both* hypotheses?

Stochastic energy-based policies

Q-function: $Q(\mathbf{s},\mathbf{a}) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$





$$\pi(\mathbf{a}|\mathbf{s}) \propto \exp(Q(\mathbf{s},\mathbf{a}))$$

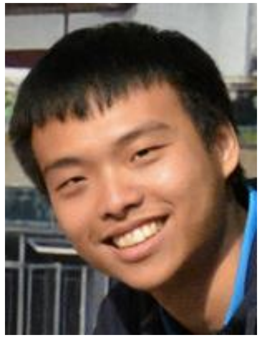
$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t,\mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t,\mathbf{a}_t))$$

$$Q_t(\mathbf{s}_t,\mathbf{a}_t) = r(\mathbf{s}_t,\mathbf{a}_t) + E[V_{t+1}(\mathbf{s}_{t+1})]$$

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t,\mathbf{a}_t))\mathbf{a}_t$$

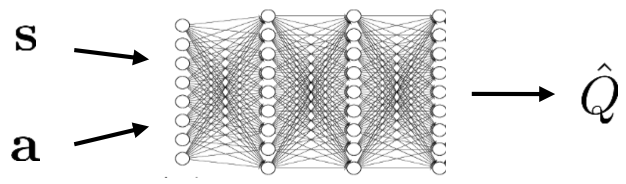
Tuomas Haarnoja

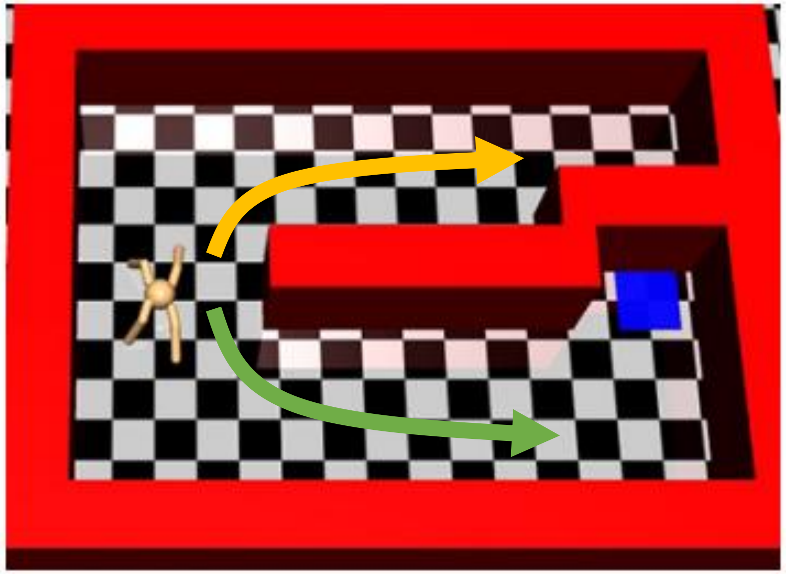
Haoran Tang



Soft Q-learning

Learned (neural network) Q-function: $Q_\theta(\mathbf{s}, \mathbf{a})$





Q-learning: $\theta \leftarrow \theta + \alpha \nabla_\theta Q_\theta(\mathbf{s}, \mathbf{a})(r(\mathbf{s}, \mathbf{a}) + \gamma V(\mathbf{s}') - Q_\theta(\mathbf{s}, \mathbf{a}))$

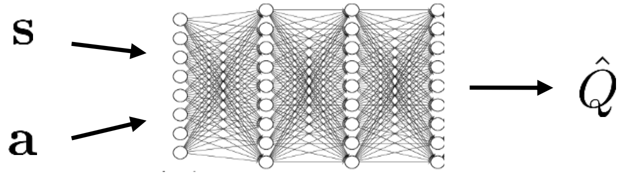
target value: $V(\mathbf{s}') = \max_{\mathbf{a}'} Q_\theta(\mathbf{s}', \mathbf{a}')$

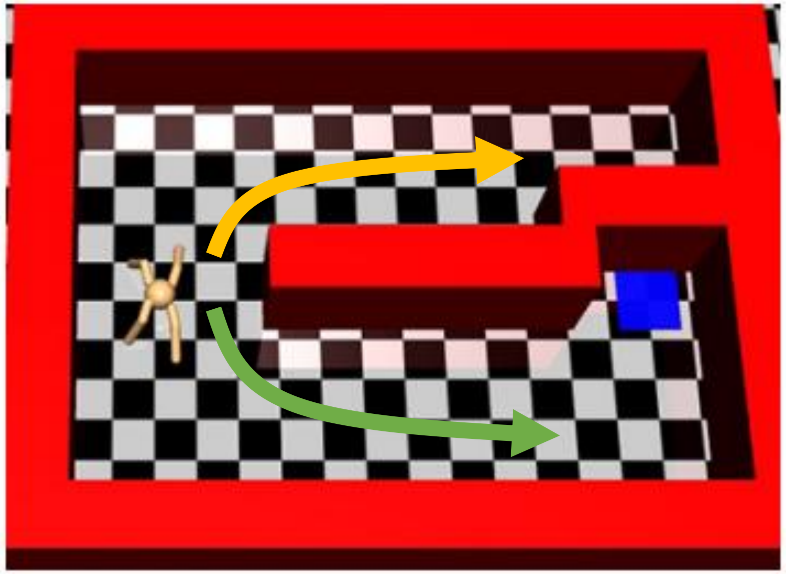
soft Q-learning: $\theta \leftarrow \theta + \alpha \nabla_\theta Q_\theta(\mathbf{s}, \mathbf{a})(r(\mathbf{s}, \mathbf{a}) + \gamma V(\mathbf{s}') - Q_\theta(\mathbf{s}, \mathbf{a}))$

target value: $V(\mathbf{s}') = \text{soft max}_{\mathbf{a}'} Q_\theta(\mathbf{s}', \mathbf{a}') = \log \int \exp(Q_\theta(\mathbf{s}', \mathbf{a}')) d\mathbf{a}'$

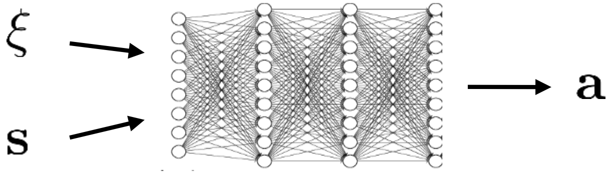
Tractable amortized inference for continuous actions

$\pi(\mathbf{a}|\mathbf{s}) \propto \exp(Q(\mathbf{s},\mathbf{a}))$





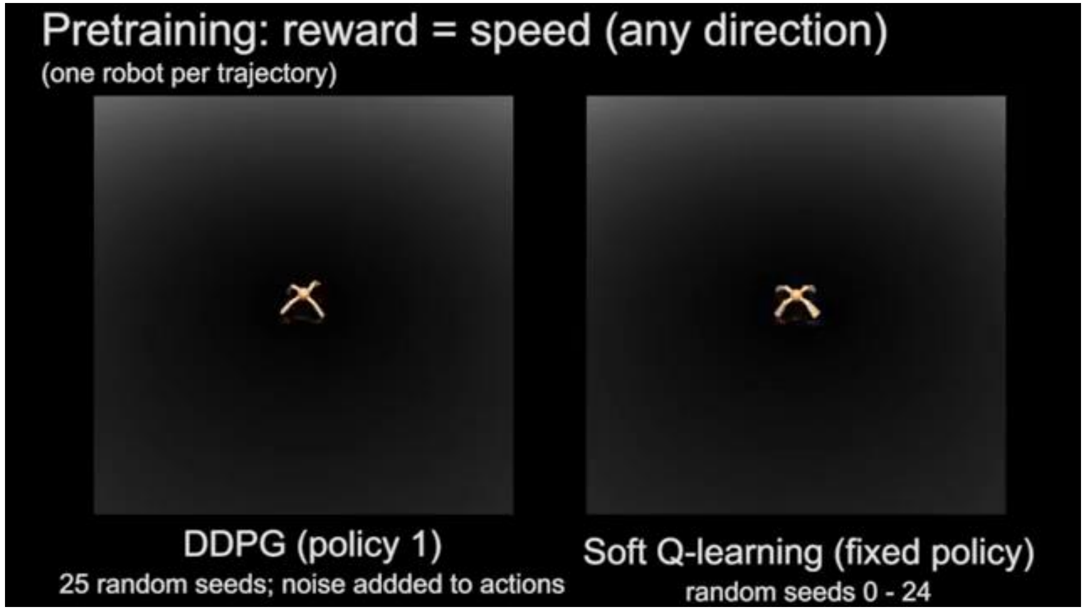
stochastic network:

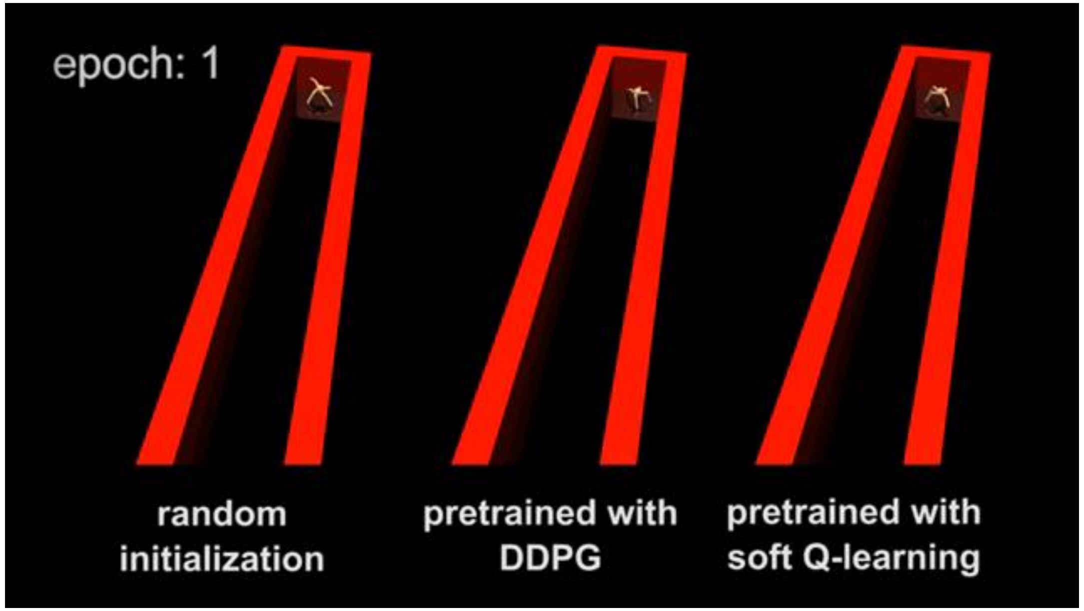


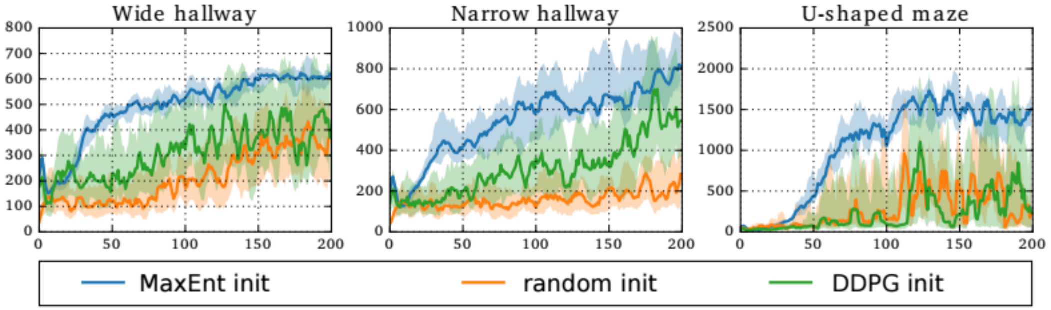
Trained with amortized SVGD to match $\pi(\mathbf{a}|\mathbf{s}) \propto \exp(Q(\mathbf{s},\mathbf{a}))$

Wang & Liu, '17

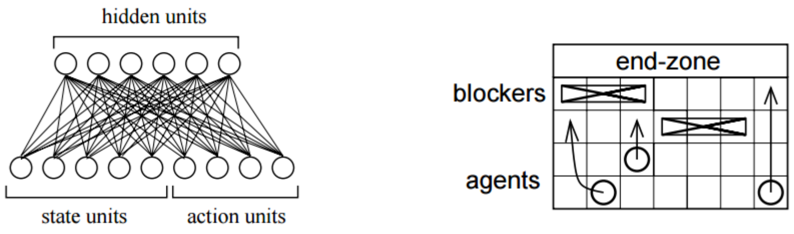
Stochastic energy-based policies provide pretraining







More work on maximum entropy policies



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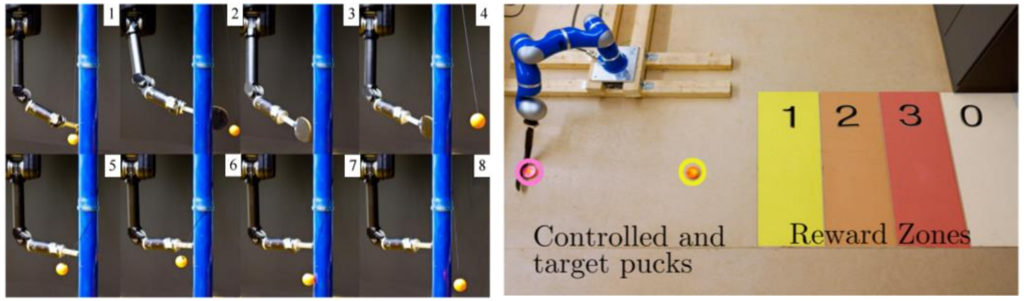

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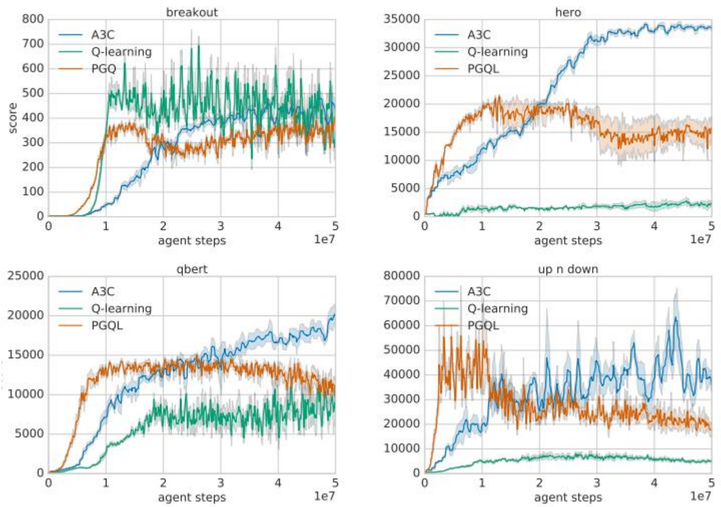

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Soft optimality suggested readings

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- Todorov. (2008). General duality between optimal control and estimation: primer on the equivalence between inference and control.
- Kappen. (2009). Optimal control as a graphical model inference problem: frames control as an inference problem in a graphical model.
- Ziebart. (2010). Modeling interaction via the principle of maximal causal entropy: connection between soft optimality and maximum entropy modeling.
- Rawlik, Toussaint, Vijaykumar. (2013). On stochastic optimal control and reinforcement learning by approximate inference: temporal difference style algorithm with soft optimality.
- Haarnoja*, Tang*, Abbeel, L. (2017). Reinforcement learning with deep energy based models: soft Q-learning algorithm, deep RL with continuous actions and soft optimality
- Nachum, Norouzi, Xu, Schuurmans. (2017). Bridging the gap between value and policy based reinforcement learning.
- Schulman, Abbeel, Chen. (2017). Equivalence between policy gradients and soft Q-learning.